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AN INTRODUCTION

TO THE

STUDY OF PHYSICS

AN INTRODUCTION
TO THE
STUDY OF PHYSICS

VOL. I
GENERAL PHYSICAL MEASUREMENTS

BY

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PREFACE

This volume contains part of a course of work in elementary physics which we have found useful in our own teaching.

It is neither a text-book of theoretical physics nor a laboratory manual of practical physics. We have sought rather to encourage inquiry than to satisfy curiosity, rather to suggest experiments than to prescribe exercises.

Every teacher of physics knows that many experiments cannot, under ordinary conditions, be made to yield satisfactory results in the hands of beginners. In dealing with such cases we have not hesitated to describe both experiment and results, introducing our description, as a rule, with one or more simpler experiments embodying the principle which it is sought to enforce. We believe that clear ideas are more easily attained by this means than by the hazardous device of manipulating imperfect observations until they seem to illustrate or "prove" some particular truth.

While therefore following, in the main, a "heuristic" plan, we have not hesitated to modify it whenever the attempt to stimulate the interest and ingenuity of the student by such means seemed likely to weaken or destroy his regard for truth and accuracy.

No attempt has been made to introduce all that might profitably be taught to a beginner. We have preferred to deal somewhat fully with certain necessary ideas, and so to arrange the book that almost every section and exercise can easily be expanded, at the discretion of the teacher, to meet the special needs of his own class.

Accordingly we have dealt chiefly with the fundamental units, length, mass, and time, and the derived units, area, volume, and density. Other derived units, such as those of force and pressure, have of necessity been introduced to some extent, but they have been used mainly for the indirect measurement of properties expressed in terms of one or other of the simpler units named above.

Two of the chief difficulties which oppose the teaching of elementary physics as a school subject we have tried to meet as far as possible. On the one hand the provision of suitable apparatus usually involves considerable expense; on the other hand its preservation requires constant care and attention. As to the former, we have sought to indicate the simplest and cheapest forms consistent with good work, and have given details of the construction of many pieces of apparatus which should enable a student, sometimes unaided, sometimes with a little supervision and help, to prepare them for himself. As to the latter, we have tried, in the instructions for the use of each instrument, to point out in what ways it is specially likely to be damaged, and to advise the student how to avoid them.

To many friends who have given us their advice and help we desire to offer our thanks.

We commend the book to our fellow-teachers in the hope that it will be useful to them, and we shall welcome any suggestions which those who may use it are kind enough to offer us.

A. F. WALDEN.
J. J. MANLEY.

OXFORD, *December* 1900.

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AN INTRODUCTION TO THE STUDY OF PHYSICS

CHAPTER I

INTRODUCTION

WE are constantly being reminded, by the reports that we receive through our various *senses*, that we are surrounded by a number of different *objects* or things.

These different objects convey, through our senses, various different impressions to our minds, and we express this by saying that they have different *properties*.

The sum of all such objects is called *nature*.

The study of nature is called *Science*, or, more exactly, *Natural Science*.

Natural science, then, is concerned with the various objects with which our impressions make us acquainted. It seeks to describe and to compare those impressions, and, from them, to judge of the objects which produce them. To do this, it is not necessary to know how it comes about that objects are able, through our senses, to make these impressions on our minds. That is an interesting and important question, no doubt, but for scientific purposes it is enough to know that the impressions are received.

The number of different objects in nature is so large, and their properties are so many, that no one can pay attention to them all. Accordingly, natural science is divided into many parts, which differ in the kind of objects studied and in the various properties of them which are considered. Some people choose, for example, to study *plants*, the number and shape of their leaves and flowers, the manner in which their seeds are formed and distributed, and so on. This part of science is known as *Botany*. Others again choose to study the positions and movements of the stars and of the sun and moon, and their special study is called *Astronomy*. This list might be made a very long one, but these examples will be enough to show what is meant by saying that science has, for practical purposes, to be divided into many parts. But all these parts have this in common, that they all deal with objects, the existence of which is perceived by the impressions which they make upon our minds through our senses. Hence, whatever part of nature we may wish to study we have to use similar methods and avoid similar mistakes.

In illustrating the *scientific method* of studying nature it will be convenient, therefore, to choose at first simple objects and to consider such properties of them as correspond to strong, simple, and easily recognised impressions. We shall see later what kinds of properties are dealt with in the branch of science called *Physics*. Meanwhile, let it be borne in mind that what we are here seeking to gain is not so much knowledge as a right method of acquiring knowledge.

Suppose now we say that we see some object, for example a penny. What do we mean by this? In reality we mean that our minds have received a number of impressions of shape, size, colour, marking, and so on. Each of these impressions corresponds to some particular property of the penny. Thus the impression of "roundness" corresponds to the circular shape of the penny. Many other common objects, a cart-wheel, the face of a clock, or a flower-pot, also give us the impression of "roundness." Why, then, do we not confuse these objects and mistake a cart-wheel or a flower-pot for a penny? The answer is, of course, that each of the objects produces *other impressions* besides that of "roundness," and we do not judge of objects by *one* of the impressions which we receive, but by *all*. We do not, as a rule, stay to think that we have actually received a vast number of different impressions; we add them together, so to speak, in our minds, and express the result of our addition sum, usually, in a single word. This addition is done unconsciously and very rapidly, but the power of thus adding up impressions is not gained all at once, as any one who has had to do with a very young child may easily observe.

Now, it is not possible to have a scientific knowledge of any object without distinguishing accurately its several different properties. This is not quite so easy to do as might be supposed, and careful training is required before it can be accomplished satisfactorily.

We have said that in learning, through our senses, of the existence of objects we acquire a certain amount of information about their properties. We may, for instance, become aware, through the sense of sight, that there is an object somewhere in our neighbourhood, and at the same time we shall gain some idea of the size, the shape, the colour, the direction, and the distance of that object. But we shall not learn whether it is hot or cold, hard or soft, heavy or light, and so on. If we desire to know about these qualities we seek the information through some other sense. We can satisfy ourselves as to the hardness or softness, smoothness or roughness of the object by touching it, when the impressions received through the sense of touch will give us the required information. If we try to lift it we shall learn, through our muscular sense, whether it is, as we say, light or heavy.

There are two important points to be noticed about the "report" upon the object thus received.

In the first place, the details of which we become conscious are only a few out of a large, and indeed indefinite number which might be known. Suppose, for instance, we were examining one of those pieces of rock known as a lodestone or natural magnet. Our senses would certainly tell us that it was of a certain size, shape, and heaviness, that it had a dark gray or black colour and a certain lustre, and that it was hard to the touch, but we might know all this and yet remain ignorant that it had a large number of other properties, such, for example, as that of attracting to itself and being attracted by pieces of steel or iron. Our acquaintance with the properties of any object is always more or less *incomplete.*

In the second place, it happens very frequently that the details which are supplied are *not exact.* Look, for example, at the two squares with circles inscribed in them shown in *Fig.* 1, holding the book as far as possible from the eye. It needs very careful inspection to convince oneself that they are the same size; yet they are. A similar, and perhaps more striking, example may be obtained by drawing two squares of exactly the same size, and ruling across one of them a dozen or so horizontal lines the same distance apart, and across the other the same number of equidistant vertical lines. The sense of sight would persuade us that both figures are oblong, the square with horizontal lines seeming to be higher than it is broad, and that with vertical lines seeming to be broader than it is high.

The sense of touch may prove even less reliable. Obtain an ordinary dried pea, or a large shot, and place it upon a smooth surface. Then cross the second finger over the first, as in *Fig.* 2, and touching the pea or shot with both fingers at once, roll it about. If we attended to our sensations of touch alone we should find it almost impossible to believe that there were not *two* peas or two shot, yet we have no hesitation in admitting that those sensations mislead us in this case.

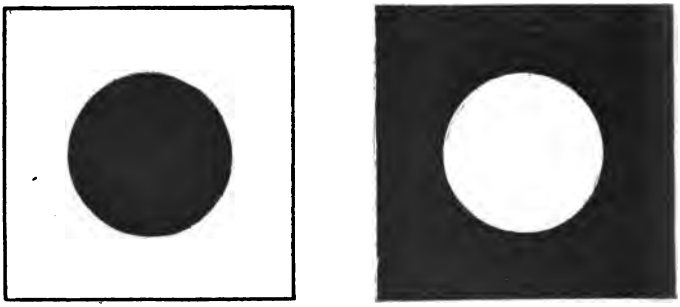

FIG. 1

We may agree then that if we wish to learn all we can about an object we must be on our guard against two things. Firstly, there is the possibility of our missing altogether some important piece of information about it. Secondly, there is the danger that even the information which we do get may be partly incorrect.

In undertaking the study of natural science then it is necessary at the outset to realise the existence of these dangers and to inquire by what means they can be avoided.

As to the former of them, it is clear from what has been said already that there is only one way of avoiding it, namely, to replace the haphazard and vague notice of an object, which in so many cases is all that we bestow upon it, by a minute, careful, and, if necessary, prolonged study of it. To this kind of notice the name of **observation** is given. But it may be necessary, before a particular property can show itself at all, that certain conditions should be fulfilled. Taking the instance named above, the power which a lodestone possesses of attracting steel or iron cannot show itself unless that lodestone is brought near to those substances. Now it might happen to us, it very likely would happen, that in the course of a minute examination of our specimen we should, accidentally as it were, bring about this state of things and discover the property. Suppose now we had done so in a particular case, and were, on some other occasion, to come across a piece of rock which in other respects, colour, hardness, and so on, resembled the lodestone we had examined. It would naturally occur to us to inquire whether this second specimen also possessed the property of being able to attract iron or steel. To satisfy ourselves upon this point we should bring a piece of iron or steel near to it, and observe whether it was attracted or not. If it were, we should class our second specimen along with our former one as having certain properties in common with it. Now it seems, at first sight, as if there were little or no difference between our discoveries of the particular property in these two cases. In both cases the discovery depends upon *observation* of the object under particular circumstances. But there is really a very important difference. The first discovery we have supposed was made more or less by accident, for though we were seeking to discover properties we were not looking for any one more than

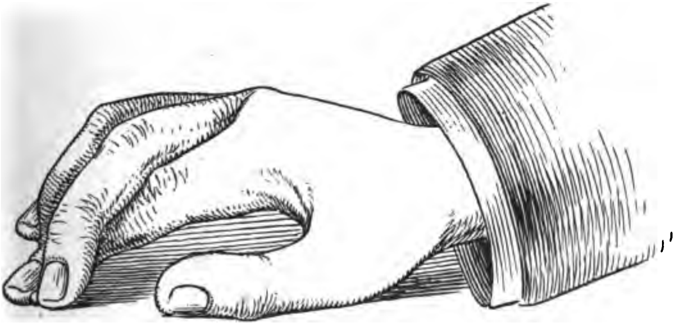

FIG. 2.

any other. The second discovery, however, was made as the result of our having deliberately brought about certain conditions in order that, if the property were there to be perceived, we might observe it. This deliberate bringing about of certain conditions in order to observe, and so make discoveries, is called an **experiment.** The making of experiments is an essential part of the scientific method of studying nature, indeed this method, as far as we have traced it at present, may be said to begin with *observation*, and to proceed from this to *experiment* and thence to *observation again.* The word experiment is often loosely used as though it included the observation which follows—as when we speak of making a discovery "by experiment,"—but this must not lead the student to forget that the experiment is the act, whatever it may be, by which we seek to render observation possible. It will be noticed that, in the case chosen above, our experiment is represented as being suggested to us by knowledge which has already been gained through observation. In other words, the experiment is made in order to answer a definite question. It is possible, of course, that the question in our minds which leads to the performance of an experiment may be quite general. What will happen, we may ask ourselves, if such and such things are done? And to find out, we perform an experiment having, it may be, no idea at all from any knowledge previously acquired what kind of result to expect. Such experiments are by no means useless, and sometimes they lead to great discoveries. Those who have made a beginning of studying chemistry will remember the famous instance of Dr. Priestley, who, about 1781, made, as he tells us, "*a random experiment*" for the amusement of some friends, and discovered the real nature of water. But such cases are rare, and most often an experiment is useful in proportion as it is designed to answer a simple definite question suggested by, and springing from, knowledge already gained.

And this brings us to another important feature of scientific method. The items of information gained by observation are not regarded merely as so many separate truths, but as portions of a connected body of knowledge, which it is hoped to make more complete. The facts which we discover may be compared with the variously shaped pieces in the so-called "puzzle-pictures." If the picture is ever to be put together, it is necessary to find all the pieces, but it is also necessary to arrange them in a particular way. And in making this arrangement, we find ourselves obliged, from time to time, to make some supposition, or, to use a Greek word, **hypothesis.** From what we have discovered, it may appear likely that certain pieces (that is, certain facts) are related to one another, and should be, so to speak, fitted together. But in doing this, we are sure to notice a gap somewhere, which needs a piece of particular shape (that is, a certain fact) to fill it. Then follows the search for this piece (the experiment by which we hope to bring this fact, if it exists, within the range of our observation). If we can find the piece (if our experiment results in the discovery of the expected fact) we may be satisfied that we are probably on the right track in arranging the pieces (that is, the facts) in the way we have begun. But if we fail to find the required piece (if the fact which emerges from our experiment fails to bear out the arrangement which our hypothesis proposes of the facts already in our possession) then that hypothesis is bad. It must be abandoned and another must be sought. It is manifest that, while the facts in our possession are few and simple, any hypothesis we may make about their arrangement is likely to be simple also. When our data increase in number and complexity, our hypotheses are likely to become more and more complex and far-reaching. If we have many facts, we shall have many possible or probable ways of arranging them. In the example chosen above the hypothesis made was that the second piece of rock would attract a piece of iron or steel. It was tested by an experiment, and the result confirmed the hypothesis. We shall return again to the subject of hypothesis and consider an example of a more complex kind in Chapter VI.

Now since the main object of acquiring a knowledge of facts is that we may be able to find the relation of one fact to others, and since, moreover, the pursuit of this object requires that we should seek our facts intelligently, in the light of what we already know, it will be seen that to make any substantial progress in the scientific study of nature a wide and accurate acquaintance with facts is necessary. And such an acquaintance can only be obtained by observation. But it must not be supposed that *every* observation upon which our knowledge of facts is based need necessarily be made by ourselves. Many people seem to suppose that the scientific study of nature is only possible so far as the student can make every discovery *for himself*. There could hardly be a greater mistake. The number of objects which come under our observation in daily life is so large, the information which we may gather about them is so varied and so complicated, that to attempt alone the study of even a small fraction of them, according to the method indicated above, would absorb all our time and all our strength. And in all human probability, the result would merely be the making of a few "discoveries" which had been made hundreds of times before. To neglect what others have already observed, the hypotheses they have made, the experiments they have performed, is as foolish as it would be for a traveller to set about the exploration of some country new to him, knowing that many travellers had been there before him, without taking the trouble to find out from the records they had left what kind of climate he would have to endure, what animals and plants he might expect to encounter, what the inhabitants were like, and what language they spoke. This independent traveller would have unexpected and unnecessary difficulties to face. He *might* add materially to our knowledge of the country whither he went; it is more likely that he would add very little. So with the independent scientific investigator who neglects the experience of those that have gone before him. The true man of science is he who strives to acquaint himself to the utmost with the discoveries of others, who tries to understand in what ways and by what means those discoveries were made, and who is ready to learn from the successes and the failures of others how rightly to conduct his own researches, and how to avoid mistakes.

Thus far we have gained some idea how the former of the two difficulties which beset the study of nature, that of imperfect recognition of properties, may be met and, to some extent, overcome. But the second, that of erroneous observation, remains, and in view of what has been said appears not less but more serious than ever. If we are thus to depend upon our own observations, guided and helped as far as possible by the observations of others, it becomes all-important that no suspicion shall attach to the correctness of those observations.

There is practically one way by which this second difficulty may be met, and that may be said to consist in a kind of *comparison*. This comparison is constantly made in everyday life. In courts of law, when some matter of fact is in question, it is customary to get as many witnesses as possible to describe what they have seen and then to place their stories side by side and compare them. That upon which they agree is believed to be true. As in everyday life, so in science, it is assumed that when several people agree about something, they are more likely to be right than any one of them by himself. It appears then that there is a certain value in having many observers of the same thing. The careful observation of any object is never useless. Even if it reveals nothing new about that object it may serve to increase our confidence in the truth of what is already known. Thus the facts ascertained by various people, being compared together, may serve to support or to correct one another.

But, it may be asked, what about the matters as to which there is disagreement? How can we decide, in case the reports differ, which is to be regarded as true? The answer is, briefly, that in a large class of cases we are guided by the majority.

For example, there are many people in the world who are what is called "colour-blind." Suppose we have two skeins of wool of certain colours and show them to, say, a hundred men and two hundred women. The experience gained from a great number of experiments shows that ninety-six of the men and one hundred and ninety-nine of the women will probably at once describe one skein as bright red and the other as bright green. But the other four men and one woman will say that both are red or both are green, though of different shades. What are we to conclude? Is there any doubt that there is some difference between the two skeins far greater than is implied in saying that they are of different shades but of the same colour? Or has not the sense of sight, for some reason, given to the five people who differ from the other two hundred and ninety-five an unusual report which we may, *for convenience sake*, describe as false? In practice we do not hesitate at all. We say there is no doubt that one skein is red and the other green, and we call those who do not see them so "*colour-blind.*"

Now in this particular case, and in many others, we recognise that the reason for the disagreement is to be found in some peculiarity, which we regard as a defect, in the organs of sense of those persons whose perceptions differ seriously from those of the majority of mankind. In such cases we can only, by comparison, find out the instances in which particular senses of certain people give false reports and put ourselves upon our guard against them. For example, men who are colour-blind are not allowed to act as drivers of locomotive engines because the colours green and red are used in signalling on railways.

But there is a far more common difficulty than this. Mistakes in observation are constantly made by people who have no peculiarity, or, as we have called it, defect of eye or other organ of sense, who can see, hear, feel, and so on, just as easily and with just the same sort of results as the rest of mankind, whose senses are quite well able, in short, to give what we regard in practice as true reports. Not long ago it was reported in the newspapers that a cyclist had been summoned and fined for furious riding in the streets of London. A number of people gave evidence, and some of them asserted that he was riding at a rate of sixty miles an hour. No doubt these men meant to speak the truth, but they were certainly talking nonsense. Such mistakes as this are really far more dangerous than those which spring from exceptional peculiarities of the organs of sense. It is not possible to pick out the people who make them and call them "speed-blind," and so on. Any one may make them; no one need do so. They are mistakes which may be avoided by training the judgment and taking certain simple precautions. One of these precautions consists in the comparison of our own impressions with those of others; but this is not enough. We have to depend here mainly upon our own efforts.

How, then, can our senses be trained to give true reports? The answer is, once again, by a kind of *comparison*. The kind of comparison needed in this case is familiar to every one. Suppose we are asked which of two of our friends is the taller. If they live in different places and are nearly of the same height we may, especially if we have not recently seen them together, have some difficulty in deciding. Even if they are in the same room with us it will probably be hard, sometimes, to say. But if we stand them back to back, and close together, we shall be able to tell much more easily and certainly. Those who have ever made such an experiment know very well that it often serves to correct a false impression. Let us now try to see in what this particular kind of comparison consists.

It will be noticed that we have supposed four things to be done. First of all, by bringing the two close together we have tried to get them *under the same conditions* so as *to avoid being misled by outside influences*, such as the height of the room, the size of the furniture in it, and so on.

Secondly, we have tried *to fix our whole attention on the one question* of height, leaving out altogether such other facts as that one is, for example, thinner than the other, and so seems taller.

Thirdly, we have tried *to get the two impressions* or reports from our senses *as nearly as possible at the same time.* This we have done because, if we get an impression of one thing and have to store it up in our minds until we get an impression of another some time after, it is extremely likely that we shall make mistakes in the comparison.

Lastly, we have *chosen the height of one* of the two *as a standard* with which to compare the other, and we express the result of our observation by saying that the height of the other is either equal to, greater than, or less than this.

Comparison of this kind, in which the three precautions named above are taken to avoid being misled, and in which the magnitude of a property possessed by one object is expressed in terms of the magnitude of that property in another object, has received a special name. It is called **measurement.** Before we can make useful scientific observations for ourselves, before we can understand how they have been made by others, before, in fact, we can make any progress in the study of nature by the scientific method, it is absolutely necessary for us to become more or less skilled in the art of measurement. Hence this volume will be found to contain chiefly some account of the methods used and the precautions taken in measuring certain properties of common objects.

As a simple instance of the value of measurement, let us take the case quoted at the beginning of this chapter of the squares with inscribed circles shown in *Fig.* 1. They seem, according to the sense of *sight*, to be of different sizes. Let us try what is the verdict of *the same sense* when it is *helped by measurement.* Lay a strip of paper, with a straight edge, cornerwise across the white square with the black circle inside it, and with a finely pointed pencil make four marks on it exactly over the corners of the square and the edges of the circle. Then take up the paper and lay it cornerwise across the black square, so that the outside mark is exactly over one corner of that square. It will be found that the other three marks also fall just over the edges of the circle and the other corner of the square. Similar measurements may be made across the middle or other corresponding parts of the squares, and it will appear, from the evidence of the very sense which at first gave us a false report, that the two figures are really of the same size.

But what are we to say of the experiment of the dried pea or shot? Even measurement will not help us there. In such a case we have first to appeal to one sense to correct another. With the fingers crossed in a particular way we seemed to *feel two* shot or two peas. The sense of *sight* tells us there is only *one.* May it not be that by crossing our fingers we have interfered in some way with the proper working of the sense of touch? We can find an answer to this question by performing another experiment. Touch the shot with both the first and second fingers, but *without crossing them.* The sense of touch now confirms the verdict of the sense of sight. The fact is, that by crossing the fingers we put the sense of touch at a disadvantage, so that it could not help conveying to us a false impression.

This indicates another source of possible error in carrying out our measurements. Fortunately it very rarely happens that all our senses are placed at a disadvantage at once, and so by the exercise of common care it is possible to avoid the mistakes which might arise from this cause.

We may now recall briefly some of the chief points in this introduction.

We possess certain *senses*, through which our minds receive *impressions* that tell us of the existence of various *objects* or things. To the extent that these objects produce different impressions on our minds they are said to have different *properties*.

The sum of all such objects we call *nature.* The study of nature, when pursued in such a way that we acquire not only a knowledge of facts but also a knowledge of the *relation* of those facts to one another, constitutes *natural science.* This is too large a study for any one man. Hence it is divided into parts which have different names according to the kind of objects with which they deal, and the properties of those objects which they take into account.

Our knowledge of nature depends upon *observation*, and in acquiring this knowledge we meet with two difficulties. On the one hand, we are always apt to overlook some of the properties of the objects studied. This difficulty we attempt to meet (1) by increasing the number and minuteness of our observations; (2) by considering such facts as we have acquired from the point of view of their relations to one another, and founding *hypotheses* upon them, these hypotheses when made being tested by performing *experiments* and observing the results which follow. In forming these hypotheses we may profitably avail ourselves of knowledge accumulated through the observations of other people. On the other hand, erroneous estimates of certain properties are frequently formed. This difficulty we seek to overcome by *measurement.*

In making measurements—

(i.) The objects to be observed must be as nearly as possible *under the same conditions.*

(ii.) The *whole attention* of the observer must be, for the time being, *fixed upon the one property* under consideration.

(iii.) The two *impressions* to be compared must be *received as nearly together as possible.*

This book has been described as *An Introduction to the Study of Physics*, and the student may naturally ask what is meant by *Physics.* The word has two meanings. Strictly speaking, it stands for "physical science." Now "physical" is only the English form of a Greek word which means "natural," and so physical is simply another name for natural science. There is no object and no phenomenon in nature which does not come within the scope of physics understood in this sense. But it has also, gradually, acquired a narrower meaning. It is applied to one of the many parts into which natural science has, in course of time, come to be divided. Even in this narrower sense, however, physics still includes within its survey all the objects in nature, only these are studied from a particular point of view, namely, that we seek to observe and to classify (or arrange) those properties which they *all have in common.* Properties which are possessed by some only among the objects perceived in nature do not primarily concern us in physics; they are left to be dealt with in groups by other sciences. Thus the *special* properties of plants form the subject of study in botany, the *special* properties of animals that in zoology, and so on. The *general* properties of all objects are the concern of physics. Now it is found, on investigation, that there are certain general (or universal) properties which may be recognised in all objects whatsoever. We cannot, without unduly extending this chapter, enumerate them here. Some of them are discussed in the remaining chapters of this first volume. Others will be introduced in the second. Meanwhile the student is asked to accept their existence as a fact. If, however, it be admitted that all objects possess certain properties in common, it is natural to suppose that they also have in common something which possesses and displays those properties. This something we call *matter*, and we assume that all objects are composed of matter. We may therefore describe physics, in its narrower sense, as that part of natural science which is concerned with the *properties of matter*.

CHAPTER II

EXTENSION IN SPACE

§ 1. UNITS.—We have just seen that *measurement* implies a very careful and accurate kind of *comparison*. Now comparison means simply *putting side by side*, so it is clear that in making measurements we shall always have to do with *at least two objects*. One of these will evidently be the thing to be measured, and the other must, it is plain, be some other object which also has the particular property that we are thinking about. But this second object is specially important because, in telling what we have found out about the first, we always refer to it, and so it becomes our *standard*. We may say, for example, that the object to be measured is so many times as big, or so many times as valuable, as the standard. Now we are quite free to choose any object we like as our standard, provided only that it has the property we are considering, but it is clear that there would be great gain if matters could be so arranged that every one would choose the same standard for measuring the same property. This is exactly what we try to do in everyday life. Suppose we were describing the value of some object, say a bicycle, we should not think of saying that it was worth five sewing machines or twenty cricket bats, because a sewing machine or a cricket bat, although each has some value, is not a convenient nor a recognised standard of value. Were we to say that the bicycle was worth nineteen sovereigns, we should be at once understood, because the coin called a sovereign is the recognised standard of value in this country. Keeping for the moment to this property of value, we might find that a sewing machine was worth three and four-fifths sovereigns, a cricket ball a quarter of a sovereign, and so on. The value of any object can then be clearly stated by stating *some number* and then giving the name of the standard used. Suppose we tried to express in this way the value of the coin chosen for the standard, we should say that it was worth *one* sovereign. The sovereign is therefore called the **unit** of value (unit from Latin *unus*, *one*).

So for every property that we wish to measure we shall have to decide upon *a unit*, and we shall describe the degree to which any object has that property by *some number followed by the name of the unit.*

Now value is a property much too difficult for us to measure, for it depends upon a large number of other properties, and that in ways which are hard to understand. Our first aim in learning how to measure will be to answer the question what property shall we choose? Our second aim will be to decide what unit we shall use in measuring it.

§ 2. CHOICE OF A PROPERTY TO BE MEASURED.—If we think of a number of common objects, a book, a table, a piano, and so on, we shall easily agree that they all *take up a certain amount of room*, or, to put it in another way, *fill a certain amount of space.* Here is a property which a little thought teaches us belongs to all objects whatsoever. There is no object so small but does fill some space or take up some room. This property of taking up room then might very well be chosen for measurement, and we shall so choose it. But the name we have given to it is at best very clumsy, and we will first try to find a better one. When we say that anything takes up room we mean that it stretches out, or reaches, or *extends* from one place to other places, and hence the property is spoken of as *extension in space.* We may say, then, that all objects possess the property called extension in space, and we will now consider how this is to be measured.

§ 3. EXTENSION IN SPACE.—We must first look at this property a little more closely. Every one who has learned any Geometry, especially as it was taught by Euclid, will remember that in the "Definitions" at the beginning we are asked at once to imagine something called a point, and are told that *a point is that which has position but no magnitude.* A point, then, is something imaginary which has no extension in space. We are next asked to imagine a *line* which, we are told, *has length without breadth.* A line, then, has *extension*, for its extremities are points, and it does stretch out from one of these points to the other, but we recognise that it has this extension *in only one direction*. From a line we pass to a *surface*, which has *length and breadth but no thickness*. A surface is thus imagined as having *extension in two directions*. And here Euclid pauses. His first four books are taken up with a discussion of the properties of points, lines, and surfaces.

We know very well, however, that we cannot actually mark or picture a point without making a dot which certainly has both length and breadth (though they are very small), and in addition has some thickness, for it is made by laying some of the "black-lead" of our pencil upon the top of our sheet of paper or a little layer of chalk upon the surface of our blackboard. So also any line which we can draw has some breadth and some thickness as well as length. Thus we cannot, so to speak, get away from length, breadth, *and* thickness except in imagination, and we realise that any object which we can see or touch *has extension in space in three directions*. Moreover, when we know all about the *length* and the *breadth* and the *thickness* of an object, we know all that there is to be known about its extension in space. It is therefore plainly important to realise exactly what we mean by these three words.

§ 4. LENGTH, BREADTH, AND THICKNESS.—The better to understand this, we will consider a very simple object, say a well-made wooden matchbox properly shut. We see at once that it has length, breadth, and thickness. But on considering it more carefully it is also seen to have six faces, twelve edges, and eight corners. Each of the twelve edges extends from one of the eight corners to another, and along each edge two of the six faces meet one another; each of the six faces is bounded by four of the twelve edges; each of the eight corners is shut in by three of the six faces, and has three of the twelve edges ending in it. The matchbox is not quite such a simple object as it appeared at first. We are led to look upon it as being bounded or enclosed by no less than six *surfaces* of the kind called in Geometry *plane* and in common language flat. Now what we call the length, breadth, and thickness are simply the distances in straight lines and in certain directions between opposite pairs of these surfaces or planes. The distance between the two planes called the *ends* of the box is the length, the distance between the two planes called the *sides* (or the *back and front*) of the box is the breadth, and the distance between the two planes called the *top and* *bottom* is the thickness. We have now to decide the exact directions in which these are to be found. About finding the length there is no difficulty. We decide which pair of opposite faces shall be called the ends of the box and on one of these ends we choose a corner. The distance between this corner and the *corresponding* corner on the opposite end is the length. If our match-box be perfectly made it will be the same whichever of the four corners at one end we choose to start from. It will be seen that all we have done has been to find the distance (the practical method of doing this is described below) between *a point* in one of the end faces and *a corresponding point* in the other. The reason for choosing a corner is simply that it is easy to recognise the corresponding corner at the other end. The breadth we might expect to find in the same way by finding the distance from a corner of the front face to the corresponding corner of the back face. But it is very likely that our match-box will not have exactly "square" corners, and it is quite plain that the distance from front to back will seem to be greater if it is found on the slant than if it be measured quite straight across. We may draw as many straight lines as we please from the front to the back edge and the length of them will depend on their direction. (This should be proved by experiments after doing Exercise IV.) To avoid confusion it has been agreed that the *breadth* shall always be found along a straight line drawn *at right angles to* the direction of *that straight line which gives the length.*

So with the *thickness*—it is the distance from top to bottom found along a straight line drawn *at right angles to both of those which give the length and breadth.*

Thus in finding the length, breadth, and thickness of an object we are free to choose *only* the direction in which we will find the length. Once this is chosen the directions in which the breadth and thickness are to be found are fixed, they are the two directions at right angles to this one. The choice of the direction in which the length is to be found is usually decided by the shape of the object, and many examples will be found in this and the succeeding chapters.

The length of an object is thus its extension in one chosen direction, its breadth is its extension in a second direction at right angles to this, and its thickness is its extension in a third direction at right angles to both of the former.

§ 5. UNIT OF LENGTH. — The units of length used in different countries are, generally speaking, different. The two with which we shall have to do are (1) the British Unit called the **Yard**; (2) the French Unit called the **Metre.** In order that there may be no doubt about the size of these units they are carefully defined by law.

The *Yard* was thus defined by an Act of Parliament passed in 1855. The straight line or distance between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the Office of the Exchequer shall be the genuine standard yard at 62° Fahrenheit, and, if lost, it shall be replaced by means of its copies.

The *Metre* was defined by a law of the French Republic in 1795 as the distance between the ends of a certain rod of platinum made by Borda, the rod being at the temperature of melting ice (0° Centigrade).

Two points will be noticed about these definitions. In the first place, it will be seen that the temperature at which the standard is to be used is specially stated. This is because the length of a metal bar becomes greater as it is made hotter and less as it grows colder. The temperature 62° F. (nearly 16°.5 C.) is about the average warmth of a room or laboratory. The temperature of melting ice (32° F. or 0° C.) is chosen because it is easily and rapidly obtained when desired. Secondly, it will be seen that two distinct ways of observing a distance are here laid down. For the *Yard* we depend on observing by means of the *eye* the distance between the centres of two ruled *lines*. We have already learned that the eye is liable to be deceived and shall be prepared to find difficulties in this **line-measurement.** For the *Metre* we depend on observing by means of the sense of *touch* the distance between the *ends* of a solid bar. Now it has been found that the sense of touch is more delicate than the sense of sight, for we can feel a difference that we could not easily see, and it would seem as if the method of **end-measurement** were to be preferred to line-measurement. However, with proper precautions, line-measurement can give very accurate results, and it is preferred to end-measurement in dealing with the standard because, among other difficulties, the ends of a bar which has to be touched in order to measure it become worn, and so the bar gradually gets shorter.

§ 6. HISTORY OF THE UNITS OF LENGTH.—The reasons which have led to the use of these units of length are very interesting, but it would take too long to describe them here.

The Yard, as defined in 1855, was intended to be a faithful reproduction of an Old English standard used by the Saxons before the Norman Conquest. Various attempts were made to fix true standard yards in the times of Henry VII., Elizabeth, and George IV. In 1834 the standard was lost through a fire in the Houses of Parliament, and a committee was appointed some time after, under Mr. Airy, then Astronomer-Royal, to reconstruct it. The present yard was the outcome of their labours, and it is not likely to be lost, for many copies are kept in various places.

The Metre was originally intended to be one ten-millionth part of the distance measured along the surface of the earth from the Equator to the North Pole, that is to say one ten-millionth of the earth's quadrant. However, the measurements made by the French men of science — carried out between Dunkerque and Barcelona more than a hundred years ago—have since been proved not to have been quite exact, so the metre is no longer defined as the ten-millionth part of the quarter-circumference of the earth, but merely as the length of the platinum rod meant by its makers to represent that distance.

Those who are interested in the history of these, and other units to be described later, should read Mr. Chisholm's book *On the Science of Weighing and Measuring, etc.* [Nature Series—Macmillan.]

§ 7. RELATION BETWEEN YARD AND METRE.—The yard and the metre are not of the same length—the metre is longer than the yard. We shall shortly (in Exercise V.) have to compare the two so as to be able to express distances measured with one unit in terms of the other. For the present it will be enough to state that *one metre* is almost equal to *one yard and a tenth* (see §§ 16 and 19).

§ 8. MULTIPLES AND SUBDIVISIONS OF THE YARD AND THE METRE.—The yard or the metre is not always a convenient standard of reference for measuring lengths. If the length to be measured is very small its value expressed in these units will be an inconveniently small fraction, if it is very large the number which expresses it will be uncomfortably big. Hence it is desirable to have other units *of the same kind*, but of different sizes, and these are obtained by multiplying or by subdividing the standard.

The yard is divided into *three* equal parts, each called a *foot*, and each of these is subdivided into *twelve* equal parts each called an *inch.*

So we have

1 YARD = 3 *feet* = 36 *inches.*
1 *foot* = 12 *inches.*

The multiples of the yard in common use are

5.5 *yards* = 1 rod or *pole.*
220 *yards* = 40 poles = 1 *furlong.*
and 1760 *yards* = 8 furlongs = 1 *mile.*

The metre is, more simply, multiplied and divided always by *ten.*

It is divided into 10 *decimetres*, each of these is subdivided into 10 *centimetres*, and each of these into 10 *millimetres.*

1 METRE = 10 *deci*metres = 100 *centi*metres = 1000 *milli*metres.
1 *deci*metre = 10 *centi*metres = 100 *milli*metres.
1 *centi*metre = 10 *milli*metres.

The multiples are

10 METRES = 1 *deca*metre.
100 metres = 10 *deca*metres = 1 *hecto*metre.
1000 metres = 100 *deca*metres = 10 *hecto*metres = 1 *kilo*metre.

It should be noticed carefully that the prefixes *deci-*, *centi-*, and *milli-* (derived from the Latin names for the numerals 10, 100, and 1000) are used for the *smaller* units obtained by dividing and subdividing the metre, while *deca-*, *hecto-*, and *kilo-* (derived from the Greek for 10, 100, and 1000) are used for the *larger* units obtained by multiplying the metre.

The advantages of this decimal system or, as it is always called, *Metric System* are plain. If we wish to express a length measured in one unit in terms of another we need only shift the position of the decimal point.

Thus the same length may be expressed as—

0.00576 kilometres 0.0576 hectometres 0.576 decametres 5.76 METRES.
57.6 decimetres 576. centimetres or 5760 millimetres.

In this book we shall nearly always use the metric system, partly for the sake of convenience in calculation, partly for reasons to be explained later, but most because it is that now employed by men of science throughout the world.

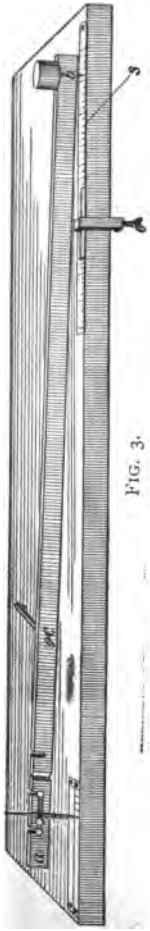

FIG. 3.

§ 9. STATEMENT OF UNIT.—We shall find that most of the lengths with which we have to do in the laboratory are such that it is convenient to express them not in metres but in *centimetres*, and thus in most cases *our* unit of length will be the centimetre, though sometimes, in measuring very small distances, we shall use the *millimetre*. It is of the utmost importance that *the unit used should be clearly stated*, and it is better always to write it after every number expressing a length. The only other plan is to write at the top of any set of results: "All lengths are expressed in centimetres" (or millimetres, etc. as the case may be), but it is easier to avoid mistakes by stating the unit each time.

§ 10. COPYING THE STANDARDS OF LENGTH.—It is evident that there can only be one real standard, and that by suitable means copies of this must be made for ordinary use. We shall now see how, taking one of these copies, we may make copies of it for ourselves.

Exercise I.

To make a copy of a standard scale.

Apparatus.—Steel rule divided into centimetres and millimetres.
Piece of cardboard of convenient size (about 40 *cms.* long).
Linear protractor (*Fig.* 3), clamps, drawing-pins.

§ 10*a*.—**Description of a Linear Protractor.**—This instrument consists of a rod of deal, *a*, *b*, about 125 *cms.* long by 1.5 *cms.* wide and 2.5 *cms.* deep, having on its under side, near the end *b*, a fine steel point about 1 *cm.* long, above which is a heavy leaden weight kept in position by an iron peg. At the end *a* three slots about 0.5 *cm.* wide and 1 *cm.* deep are cut in the top of the rod so that their centres are 2.8 *cms.* apart, and on the front face of the rod two V-shaped grooves are made midway between the slots, one about 0.5 *cm.* and the other about 0.1 *cm.* deep. In the larger of these grooves an ordinary lead pencil (best one of the hard kind marked H H H), sharpened to a chisel-point, can be held firmly in position by an ordinary screw clamp. The smaller groove serves to hold similarly an ordinary drawing-pen or a fine steel point as required. At the place marked *c* a conical rod about 40 *cms.* long, furnished at the farther end with a blunt projection pointing downwards, and about 1 *cm.* long, passes through the larger rod and prevents it from tilting over. The length of the projection on the lateral rod, and the projecting part of the pencil should be so arranged that when the instrument rests upon these and the steel point it is as nearly as possible level. Finally, a roller about 6 *cms.* in diameter (a glass bottle will serve) is provided, upon which the end toward *a* can rest when the pencil is not in use.

Method.—The steel scale, *s*, is clamped along the edge of the bench. About 100 *cms.* farther to the left a piece of cardboard upon which three horizontal lines have been ruled, as in *Fig.* 4, is fixed by means of drawing-pins so as to be in a line with the scale. The steel point of the linear protractor is placed in one of the *cm.* divisions of the steel scale so as to be about opposite the middle of the small (*mm.*) divisions, the other end of the instrument resting upon the roller. The end *a* is then raised clear of the roller, and the pencil (or drawing-pen) brought down on to the topmost of the three ruled lines. It should fall somewhere near the right-hand end of the cardboard. Then it is moved firmly forward to the edge of the card, raised, and replaced upon the roller. The steel point is now lifted and placed in the next *mm.* division to the right. The pencil is this time brought down upon the lowest of the three ruled lines, and again drawn forward to the edge. These operations are repeated till the fifth *mm.* division is reached, when the mark is made from the second line. At the tenth division it is again made from the topmost line. In this way the *cms.* and *half-cms.* are clearly distinguished from the *mms.*, and with reasonable care a very accurate copy of the steel scale can be made of any desired length. About 30 *cms.* is a useful length for a cardboard scale.

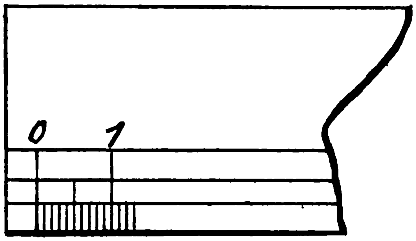

FIG. 4.

It will be noticed that the marks thus made upon the card are not really straight lines, as they ought to be, but parts of circles. However, since the radius of these circles is very large, and the marks themselves are very short, the amount of curving in them is so slight that it is practically not noticeable.

§ 11. GLASS SCALE.—For many purposes it is most useful to have a copy of the standard scale etched upon a strip of glass.

Exercise II.

To make a copy of a standard scale on glass.

Apparatus.—Steel rule and linear protractor with steel point in smaller V-groove.
Strip of common window-glass about 20 *cms.* long by 2.5 *cms.* wide.
Shallow tin plate dish large enough to contain this strip, and containing filtered paraffin wax (see § 11*a*).
Solution of hydrofluoric acid, blotting-paper, glass rod.
Strip of wood about 30 *cms.* long by 5 *cms.* wide and 0.5 *cm.* thick.
Clamps, small piece of wire, large darning-needle.

§ 11*a*. **To prepare a Bath of Clean Paraffin Wax.**—The wax is melted in a porcelain dish and stirred with a thermometer till its temperature is about 80° C. A glass funnel with a short, wide stem and large enough to contain the whole of the molten wax is warmed by turning it about in the hot air some way above a Bunsen flame, a folded filter-paper is placed in it, and it is supported just above the clean tin-plate dish. The melted wax is easily filtered, and is then ready for use.

Method.—The sharp edges of the glass strip are ground either by rubbing them in a mixture of emery powder with turpentine in which a little camphor has been dissolved, placed upon a piece of plate glass, or, more easily, by means of a "carborundum" file. The glass is then thoroughly washed with soap and water, then with clean water, and dried with a clean white cloth.

The filtered wax is just melted, and the cleaned glass placed in the dish. After it has remained for about a minute one end is raised out of the bath by means of a wire hook, the *edges* seized with the thumb and finger, and the whole slowly raised into an upright position just clear of the molten wax. This allows the melted wax to drain off, leaving only a very thin coating on the glass. Then while still warm it is laid upon the strip of wood. The wax in solidifying cements the glass on to the board. This is then clamped on to the edge of the bench so as to be in about the same position as the cardboard in Exercise I. Three horizontal lines are now ruled upon the waxed surface with the point of the needle, care being taken to press firmly down upon the surface of the glass.

Then the graduation is carried out exactly like that of the card in Exercise I. It will be found best to make a scale of 15 *cms.* We have now uncovered certain parts of the glass exactly 1 *cm.*, 1 *mm.*, etc. apart, and it only remains to mark these uncovered parts. This is done by taking advantage of the fact that a solution of hydrofluoric acid in water attacks glass and does not attack wax. A piece of blotting-paper, the same size as the glass strip, is laid upon the latter, and the hydrofluoric acid solution is carefully dropped upon it so as thoroughly to wet it.

[This should be done over a sink, and great care should be taken not to let any of the hydrofluoric acid touch the fingers.]

The wet strip of blotting-paper is gently moved to and fro over the waxed surface by means of a glass rod (covered at the end with a piece of narrow india-rubber tubing to prevent its slipping) for three minutes. Then the paper is washed off under the tap, and the glass loosened from the wood by pouring hot water over it. The glass scale is then cleaned by rubbing it with a rag moistened with turpentine. It is, however, still desirable that the graduations should be numbered, and it is better (for reasons explained in § 13) that these numbers should be on the opposite side of the strip to the graduations. The strip should be again washed, dried, coated with wax, and laid upon the board *face downwards*. The graduation to the left should be marked 0, the next *cm.* division 1, and so on. The etching in of the numbers is performed as already described, and the scale is finally cleaned ready for use.

This making of scales is a little troublesome, but the success of many of the experiments which follow depends upon the care and accuracy with which it is done. For this reason very full directions have been given, and the student is urged to take great pains in preparing his own scales. In the next chapter we shall show how they are to be used.

CHAPTER III

MEASUREMENT OF DISTANCE

§ 12. USE OF SCALES.—The scales which have been made in Exercises I. and II. are now to be used for the purpose of carrying out *line-measurements*, that is comparisons in which we place the scale close to the object to be measured and judge by means of the *eye* the relation between them.

Exercise III.

To measure the distance between two points.

Apparatus.—Cardboard scale prepared in Exercise I. (§ 10).—Paper or cardboard, drawing-pen, ruler.

Method.—Two small crosses of fine lines are ruled upon the paper at a distance not greater than the length of the scale. The scale is placed *upright*, as in *Fig.* 5, with the zero division exactly over the point where the two lines at *a* cross one another. Make the scale pass exactly across the point where the lines at *b* intersect. Read off the number of *cms.* and *mms.* to the division mark which *most nearly* falls over *b* (see § 14 for more exact measurement).

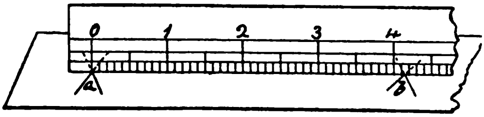

FIG. 5.

§ 13. WHY THE SCALE IS PLACED UPRIGHT.—Take two objects, a couple of ink-pots will do, and place them about a metre apart upon a table. Then close one eye and, with the other, look along the surface of the table so that the further object is just hidden behind the nearer. Then move the head *a little* to the right or left. It will be noticed that both objects can now be seen, and that on moving the head from right to left and back again the nearer object *seems to move* in the opposite direction relatively to the further one. This apparent movement is *greater the farther the objects are apart* (*try it*), and must be familiar to any one who has looked out of the window of a railway train in motion. The seeming movement or *displacement* of a thing observed when the position of the observer is shifted is called **Parallax** (from a Greek word meaning alteration or change—in this case, of position). The closer two objects are together the less does the position of one appear to change with reference to the other when the eye of the beholder is shifted. Hence the scale is placed upright so that its graduations may be as close as possible to the marks upon the paper. Were it laid flat there would be the thickness of the cardboard between them, and the least shifting of the eye to either side would produce *parallax* (*try it*) so that it would be difficult to tell which division fell exactly over *b*.

This parallax is a frequent cause of mistakes in measuring, and it will be seen that we have constantly to guard against its occurrence.

§ 14. USE OF GLASS SCALE.—We shall now see how, by means of our glass scale, we may avoid the error due to parallax still better.

Exercise IV.

To measure the distance between two points.

Apparatus.—The glass scale made in Exercise II.
Paper or card marked as for Exercise III.

Method.—The scale is laid *face downwards* upon the paper (why? see § 13) so that both crosses fall about the middle of the *mm.* divisions. The zero line is made to fall exactly upon the point *a*. Then the position of the other cross is carefully noticed. It will very likely not fall exactly beneath one of the *mm.* divisions, and we shall therefore be obliged to judge as well as we can where it would come supposing each of the *mm.* spaces were further divided into ten equal parts. In the case shown in *Fig.* 6 the cross lines intersect between the divisions 4.4 and 4.5 and somewhat nearer to the 4.4 than to the 4.5. The length is therefore greater than 4.4 and less than 4.45. It would in the case given be judged as 4.43.

§ 15. ESTIMATION.—It often happens that, as in the last Exercise, the magnitude we have to observe does not *exactly* coincide with any of the magnitudes marked upon our scale. In such cases we judge as nearly as we can what division *would be* reached supposing the scale were more minutely subdivided. This process of judging is called **estimation.** It is very important and a great deal of practice is needed to do it well, as may easily be seen by trying to grasp a graduated ruler, turned face downwards, exactly in the middle, or at one third of its length, etc. (*try it*). We shall presently learn how to avoid estimation in many cases, but it is important to practise it.

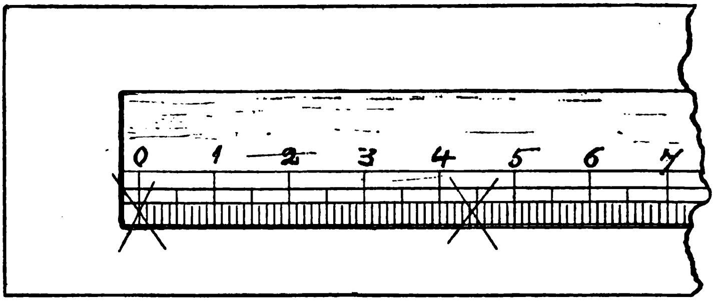

FIG. 6.

§ 16. COMPARISON OF STANDARDS.—We are now able to measure a simple length with fair accuracy, and we will proceed to compare the lengths of the British and Metric standards.

Exercise V.

To compare the lengths of the *Yard* and the *Metre*.

Apparatus.—Glass scale, ruler (best a steel one) marked in *inches* and tenths.

Method.—Measure (with all precautions suggested in Exercise IV.) the distance in centimetres between the inch divisions 0 and 2, 1 and 5, 6 and 11 (and any others you like). From the result in each case calculate the *length of an inch in cms. and fraction of a cm.*

From this calculate, in each case, the *length of a yard in cms.*

Express these lengths in *metres.*

§ 17. AVERAGE AND MEAN.—It will be noticed that in the last Exercise it was recommended that at least three separate measurements of the length of an inch in *cms.* should be made, and that each time a length of more than one inch was observed for this purpose. Now the object of our experiment was to find out the truth about the length of an inch, and there are two reasons which might have hindered our doing it had we taken only one reading. In the first place, the inch-divided scale may not be quite regular, and by making measurements in different parts of it we are able to find out if this be the case and to allow for it. In the second place, not one of our measurements is likely to be quite exact. We are almost sure to make some mistake in estimation, and the size of this mistake will probably be about the same whether we are measuring a length of two inches or of four, or of five, being, if we are careful, only a fraction of a millimetre. In calculating the length of one inch we divide, in the first case, by two, in the second by four, and in the third by five. Thus, half our probable mistake appears in the first value, a quarter of it in the second, and only a fifth of it in the third. On the whole, then, the third result would seem to be the best. But, again, it is probable that sometimes we overestimate and sometimes underestimate the length, and on the whole we are nearly as likely to do the one as the other. If, therefore, we take our *three estimates* of the length of an inch, add them together, and divide their sum by *three*, we may have rather more confidence in the *average* value so found. But it is plain that even this is not quite enough, for we have seen that these three different results are most likely not equally good, and we ought to allow for that too. Now this can be done, but it is not a very simple or easy matter, and we shall not attempt it here. If we could get it, the final result, after averaging with due allowance for the probable goodness of each separate result, would be called the **mean.** Although our simple average is thus not the best mean possible we shall henceforth speak of it as *the mean*, bearing carefully in mind that it is only a rough-and-ready one.

Henceforth, several *separate* measurements of any magnitude should always be made and the *mean* of the numbers found given as the final result. In calculating this mean it is well first to look at the series of numbers with which we have to deal, and notice whether any of them differ *very largely* from the rest, say, for example, by whole *mms.* in measurements of length. If so, they are probably the results of bad observations and should be *left out* altogether.

§ 18. THE WAY TO MAKE EXPERIMENTS.—Here a warning is necessary. In making the several measurements from which a mean is to be calculated the student should be most careful to *make each one quite separately, to let each be a single deliberate attempt to find out the truth. He should put down the number in his note-book and then try to forget it, for the time, before making a fresh reading.* Not to do this is to run a risk, which amounts to a certainty, of deceiving himself. If he allows a second estimate to be influenced by a former one he is unconsciously "cooking" his result. Now "cooking" when conscious is a kind of lying, when unconscious a form of self-deception, scarcely less dangerous. Nothing could be more unscientific, for the aim of science is the pursuit of the truth.

§ 19. RELATIONS OF BRITISH AND METRIC UNITS.—Having, then, discovered the relation between the inch and the centimetre by taking the mean of the measurements done in Exercise V., the student should use it to draw up a list expressing the inch, foot, yard, and mile in metres, and another giving the values of a centimetre in inches, a metre in yards, and a kilometre in miles. *After having done this* let him compare his results with the following tables :—

1 inch = 0.0254 metre
1 foot = 0.3048 metre
1 yard = 0.9144 metre
1 mile = 1609.0000 metres

1 centimetre = 0.3937 inch
1 metre = 1.0937 yard
1 kilometre = 0.6214 mile

and work out the following problems :—

(1) Assuming that 1 metre = 39.37 inches find the relation between 80 miles and 13 kilometres.

(2) Express 5.08 *mm.* as a decimal of 1 *in.*

(3) Mont Blanc is 15,782 feet high. Express this in metres (t[illegible] place of decimals).

§ 20. INDIRECT MEASUREMENT.—So far we have applied our scale directly to the object we wished to measure. It is not always convenient to do this. Often it is easier to take an exact copy of the distance to be measured and apply the copy to the scale. A common way of doing this is to use a pair of ordinary compasses.

Exercise VI.

To measure the distance between two points, using compasses.

Apparatus.—Paper or card marked for Exercise III. Pair of compasses, steel scale.

Method.—Place the point of one limb of the compasses upon one of the crosses and open them until the point of the other limb falls exactly upon the second cross. The distance between the points of the compasses is now an exact copy of the distance to be measured. Lift the compasses carefully and apply them to the scale. Compare the reading with that of Exercise IV.

§ 21. END MEASUREMENT.—This indirect method is very useful when we have to find the distance between the ends of an object. For this purpose, a special kind of compasses known as **callipers** is used. These are shown in *Fig.* 7. In using them the jaws are opened widely enough to admit the object to be measured and then brought together so as *lightly but firmly* to touch the ends. They are fixed in this position by turning the screw *s*, withdrawn and applied to a scale.

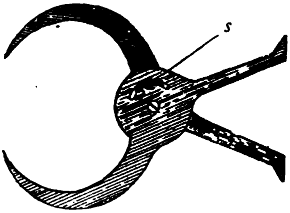

FIG. 7.

Exercise VII.

To measure the width of the glass scale.

Apparatus.—Glass scale, common callipers.

Method.—Apply the callipers to the edges of the scale, as directed above, [illegible] measure the distance between their extremities on the scale itself. Make [illegible] least six measurements at different places (why?), and find the mean.

Example of an actual experiment—

Observation.	Width.
1	2.53 *cms.*
2	2.51 ,,
3	2.50 ,,
4	2.48 ,,

Mean 2.50 *cms.*

§ 22. FINER MEASUREMENTS.—We shall now consider some of the devices by which it is possible to measure very small distances more accurately than by estimation. One of the simplest, though the least useful of these is known as the **diagonal scale.** *Fig.* 8 represents part of a rectangle exactly 1 *cm.* (or 1 *in.* etc.) wide and divided into *cms.* (or *ins.* etc.) by the vertical lines. It is divided into four equal strips by three horizontal lines, and the sides *ab* and *cd* of the end space are also divided into four equal parts and the points joined diagonally as shown. The straight line *df* evidently slopes backward through a distance of $\frac{1}{4}$ *cm.* (or $\frac{1}{4}$ *in.* etc.), since it runs from the division 9 to the division $9\frac{1}{4}$, and in so doing crosses in succession four equal spaces. In each of these spaces it advances one quarter of the whole amount, that is $\frac{1}{4}$ of $\frac{1}{4}$ or $\frac{1}{16}$. So that, for example, the distance between the two crosses marked is easily seen to be 1 *cm.* + $\frac{2}{4}$ *cm.* + $\frac{3}{16}$ *cm.* = $1\frac{11}{16}$ *cm.* Thus, although no division smaller than a quarter is made, it is possible to read to the quarter of this, *i.e.* to a sixteenth. If, therefore, we had divided into tenths in a similar way we could have read to the tenth part of one-tenth, that is to one-hundredth of a whole division. The disadvantage of this plan is that it is very hard to construct the scale accurately when the subdivisions are small, and that with a large number of lines close together, it is difficult to read.

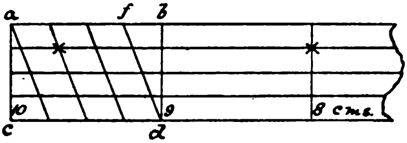

FIG. 8.

§ 23. THE VERNIER.—Although the diagonal scale is not much used, the *principle* of obtaining fine measurements by means of a sloping line is a valuable one, and we shall see in § 32 that it is applied in the instrument called a *screw.* Meanwhile, there is a very simple plan, invented in 1631 by Pierre Vernier, by which fractions of a unit may be accurately measured. This consists in the use of a small extra scale, called, after its inventor, a **vernier,** so marked that each of its divisions is exactly *a known fraction less or greater* than the divisions of the main scale. Suppose we decided that each division of the vernier should be exactly $\frac{1}{7}$ *less* than the divisions on the main scale; we should mark off a distance equal to 6 scale divisions and divide it into 7 equal parts. Each vernier division would then be equal to $\frac{6}{7}$ of a scale division. If we wished to have the divisions of the vernier $\frac{1}{7}$ *greater,* we should take a distance equal to 8 scale divisions and divide that into 7 equal parts. Then each vernier division would be equal to $\frac{8}{7}$ of a scale division. *Figs.* 9 and 10 will show how, with the help of these extra scales, we could read accurately to $\frac{1}{7}$ of our scale divisions.

The distance marked *a* is made equal to that which is to be measured by introducing the object and sliding the vernier along till it just touches. We have to find the position on the scale of the end of the vernier which is next to the object. In both diagrams, this is plainly more than 2, and less than 3, divisions from the zero of the scale, and we have to find out by how much. One way of doing this occurs to us immediately. If we look at *Fig.* 9, we see that the *6th mark* upon the vernier scale falls exactly under the 8th division of the main scale. We know, therefore, that the length *a*, *plus* the length of 6 vernier divisions just amounts to 8 scale divisions. But in this case each vernier division is $\frac{6}{7}$ of a scale division, and so 6 of them amount to $5\frac{1}{7}$ scale divisions. If, then, we take away $5\frac{1}{7}$ from 8, we get, for the *length of a*, $2\frac{6}{7}$ *scale divisions*. Turning now to *Fig.* 10, we notice that the first mark upon the vernier scale (the numbers in the figure are explained below) falls just under the 4th division of the main scale. In the same way then the length of *a*, *plus* 1 vernier division is now equal to 4 scale divisions, and 1 vernier division being in this case $1\frac{1}{7}$ scale divisions we get, taking $1\frac{1}{7}$ from 4, again $2\frac{6}{7}$ *scale divisions* as the *length of a*. This shows that whichever vernier we use, the result is the same, but this is a clumsy way of finding it, and we shall now work out an easier and better one.

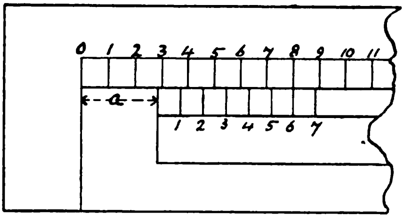

FIG. 9.

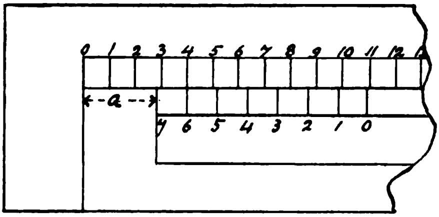

FIG. 10.

In *Fig.* 9, the 6th vernier division, counting from the end next to the object, falls exactly under the scale division marked 8. Remembering that each vernier division is $\frac{1}{7}$ *less* than a scale division we know that

the distance between the 8th scale division and the 6th vernier division $= 0$
,, ,, 7th ,, ,, 5th ,, $= \frac{1}{7}$
,, ,, 6th ,, ,, 4th ,, $= \frac{2}{7}$
,, ,, 5th ,, ,, 3rd ,, $= \frac{3}{7}$
,, ,, 4th ,, ,, 2nd ,, $= \frac{4}{7}$
,, ,, 3rd ,, ,, 1st ,, $= \frac{5}{7}$
,, ,, 2nd ,, ,, end next to the object $= \frac{6}{7}$

so that the length a is $2\frac{6}{7}$ scale divisions. Now this can be expressed very shortly. *The number of sevenths that the end next to the object is beyond the last scale division included is simply the number of the coinciding vernier division counting from the end next to the object.*

A vernier like that of *Fig* 9, in which the vernier divisions are *shorter* than the scale divisions, is called a **Number 1 vernier.**

Turning to *Fig.* 10, it is the first vernier division, counting from the end next to the object, which falls exactly under one of the scale divisions, this time the one marked 4. In this case each vernier division is $\frac{1}{7}$ *more* than a scale division, so that

the distance between the 4th scale division and the 1st vernier division $= 0$
,, ,, 3rd ,, ,, end next to the object $= \frac{1}{7}$

Here, then, we have found that the end of the vernier is $\frac{1}{7}$ behind the 3rd scale division, and therefore $\frac{6}{7}$ in front of the 2nd, so that the length a is, as before, $2\frac{6}{7}$. The difference between this case and the former is that here we are measuring backwards, and if therefore we number the vernier divisions backwards we shall find that the rule given above holds good in this case as well. We simply have to write *counting from the end farther from the object.* A vernier like this of *Fig.* 10, in which the vernier divisions are *longer* than the scale divisions, is called a **Number 2 vernier.** Notice that in a No. 1 vernier the divisions are counted in the *same direction* as those of the main scale, and in a No. 2 vernier they are counted in the *opposite direction.*

§ 24. GENERAL RULE FOR MAKING A VERNIER.—It will now be plain that the device just described makes it possible to measure lengths to any desired fraction of a chosen unit. In the case discussed the fraction was $\frac{1}{7}$, but we might just as well have chosen, say, $\frac{1}{19}$. In that case each vernier would have had 19 divisions, the *No.* 1 being made by dividing a length equal to 18 scale divisions into 19 equal parts, and the *No.* 2 by similar division of a length equal to 20 scale divisions. We can express this in a *general* statement, that is, one which is true whatever the particular number we have in mind. If it be desired to make a vernier to read to $\frac{1}{n}$ of a scale division, that vernier must have n equal divisions. The No. 1 vernier must be made by dividing the length of $n-1$ scale divisions into n parts, the No. 2 vernier by similar division of the length of $n+1$ scale divisions. The graduations of the No. 1 vernier must be numbered in the same order as, and those of the No. 2 vernier in the opposite order to, the divisions of the main scale. Except for some special reason (for example, on a scale of feet a vernier might conveniently be made to read to twelfths [why ?]) it is usual to arrange a vernier so that it reads to *tenths.* Those which we shall now proceed to make and use will be arranged on this plan.

The vernier is of the highest importance. All instruments for measuring length, except the very coarsest, are furnished with one, and it is essential that the student should understand it thoroughly. He will do well, before going further, to solve the following problems :—

(4) A vernier has 50 divisions, and the main scale divisions are half an inch in length. To what fraction of a scale division will it measure? To what decimal of an inch? How long will the No. 1 vernier be?

(5) Explain how a vernier could be made to measure to 0.01 *cm.* if the divisions of the main scale were each 0.25 *cm.* in length.

§ 25. CONSTRUCTION OF A VERNIER.—If the principles of the two previous sections have been mastered, it will now be a very easy matter to make a scale and vernier.

Exercise VIII.

To construct a scale with vernier to read to 0.1 *cm.*

Apparatus.—Piece of cardboard (about 20 *cms.* long by 6 *cms.* wide).
Strip of wood (the lid of a cigar-box will do).
Linear protractor with drawing-pen, set-square, steel *mm.* scale, clamps, drawing-pins, sharp penknife.

Method.—Three horizontal lines, *a*, *b*, *c*, exactly parallel, are drawn on the card, as in *Fig.* 11. The card is then pinned to the board, and clamped in position upon the bench, the linear protractor and steel scale being arranged as in Exercise I. Then, with the drawing-pen, the graduation is begun two or three *cms.* from the edge (*ac* in *Fig.* 11) and continued as far as convenient, *whole cms. only* being marked. The 5th, 15th, etc., graduations are prolonged a little beyond the line *b*, and the 10th, 20th, etc. carried right through to the line *a*. Next prolong the zero-line *ca*, by means of a set-square, to the edge of the card at *d*. Then with a sharp knife and the steel rule cut carefully along the lines *dc* and *ce*. The card is now pinned at *d* and *e*. With the piece cut out the *vernier* is now made, and it is well to construct both forms of vernier upon the same strip of card.

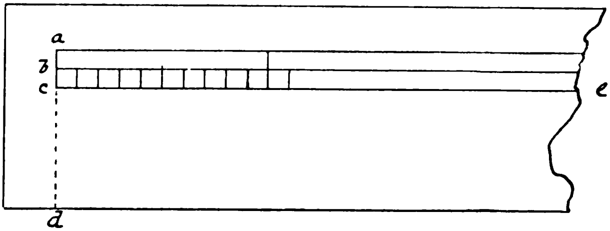

FIG. 11.

The card is pinned down in position for graduation, and two horizontal lines *ab* and *cd* are drawn across it as shown in *Fig.* 12. Then the graduation of *No.* 1 *vernier* is carried out. The zero-line *ef* is set off, the steel point of the Linear Protractor carried back *nine millimetres*, and a second line marked, and so on until *ten* divisions have been marked. It is convenient to make the fifth and tenth marks project a little beyond the line *ab*.

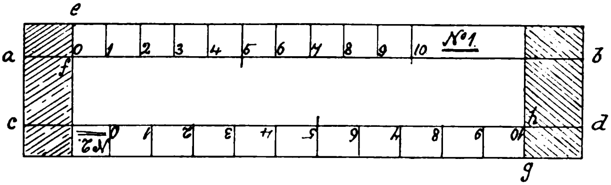

FIG. 12.

The card is now turned round, and *No.* 2 *vernier* marked on the opposite edge. The "zero" line *gh* is set off about 15 *cm.* farther along the card than *ef* (the zero line of No. 1). The steel point is then carried back *eleven millimetres* at a time, till ten divisions have been marked as before.

Now with a set-square prolong the lines *ef* and *gh* right across the card, and very carefully cut along these lines with a sharp knife, thus removing the shaded parts in *Fig.* 12.

No. 1 vernier is to be marked 0, 1, . . . 10, beginning at the outside end of the scale, No. 2 vernier 10, 9 . . . 0.

The scale and vernier so made will be much improved if glued upon suitable strips of thin wood, as in *Fig.* 13. But great care must be taken that these are exactly the right size, and that they are perfectly plane.

§ 26. USE OF VERNIER.—From what has been said, it is evident that the vernier and scale just made is intended to measure accurately to 0.1 *cm.* If carefully made, it will do so easily. Before attempting to use it, the student should test it by seeing that when the end of the vernier is pushed up into the corner of the scale, the zero lines exactly coincide, and the tenth graduation of the vernier falls exactly below (in case of No. 1), the ninth, or (in case of No. 2), the eleventh graduation of the scale [should this not be the case, the instrument is probably useless. See, however, § 29].

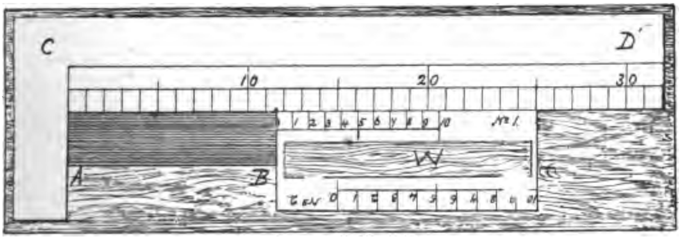

FIG. 13.

Exercise IX.

To measure the length of a block of wood by means of a centimetre scale with vernier.

Apparatus.—Scale and vernier made in Exercise VIII. Wooden blocks.

Method.—The block is placed in the corner *c* (*Fig.* 13), so that its edge lies along the scale and the vernier applied beyond it, and pushed up till its zero-line just touches the end of the block firmly. Read off the whole *cms.* from the main scale, and the tenths from the vernier (as in § 23). Number the edges of the block and measure the length of each one.

§ 27. SLIDE CALLIPERS.—We may now turn to an instrument which in principle is like the one constructed in Exercise IX., but in practice far more delicate. If instead of having made our main scale divisions 1 *cm.* long, we could have made them 1 *mm.*, then, with a suitable vernier, we might evidently have measured to 0.1 *mm.* This is what is done in a special instrument, used for end measurement, known as the **Slide Callipers.** The construction will be plain from *Fig.* 14. The callipers are made of thin steel plate, and are so arranged that the inner jaw is movable backwards and forwards along a horizontal bar fixed to the outer. Upon this fixed bar a scale of millimetres is marked, and attached to the movable jaw is a vernier scale, so placed that it slides exactly parallel to and very close upon the main scale. This vernier is of the kind known as No. 1, and is graduated so as to read to *tenths* of a *mm.* [How long will the vernier scale be? How many divisions will it have? (§ 24).] When the faces of the jaws are placed in contact, the zero of the vernier is intended exactly to fall upon the zero-line of the main scale. If the jaws be moved never so little apart, we can see that the end of the vernier has moved some way along the scale. The number of whole *mms.* is counted directly, the decimal of a *mm.* read off by observing the number of that vernier graduation which coincides with one of those on the main scale. If therefore the jaws be drawn apart, and an object be inserted between them, then by

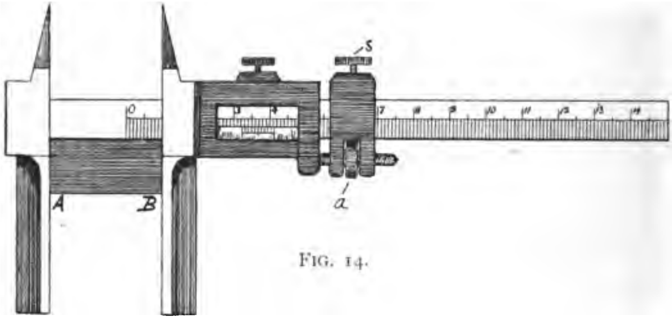

FIG. 14.

bringing the movable jaw carefully along so that its face just touches the object, we can read off directly the length of the object, as ascertained by end measurement, accurately to 0.1 *mm.*

§ 28. FINE ADJUSTMENT.—It is not easy to acquire the delicacy of touch needed to make the movable jaw come just into contact with object without pressing unduly against it. Moreover, it is important that the true position once found should not be easily lost. For these reasons, the better forms of the instrument are provided with means for steadying the final movements and rendering them very gradual. These consist of the two screws shown in *Fig.* 14. One of these, marked *s*, serves to clamp the movable jaw in position when it has been brought, by sliding it along, so as *very nearly but not quite* to touch the object. It can now only be moved by turning the other screw, marked *a*, which is so arranged that when it is turned the jaw is moved very gradually forward or back. If the milled edge of this screw be *very lightly* held between the thumb and finger it will be found that with a little practice the exact moment of contact is easily detected. An arrangement for steadying the final movements of an instrument and making them very gradual and slight is called a **fine adjustment.** All instruments intended to give accurate results need to be provided with fine adjustments, and usually, as in the present case, these depend upon the action of a screw which will be explained in § 32.

§ 29. ERROR OF INSTRUMENT—CORRECTIONS.—Before we proceed to use the slide callipers one highly important matter requires attention. Unless the instrument has been very well made it is quite possible that, when the jaws are made just to touch (which must be done carefully, with the help of the fine adjustment as described in § 28), the zero of the vernier *may not coincide exactly* with the zero of the main scale. Should this be the case, it is clear that all lengths measured with this particular pair of callipers will be found either always greater or always less than they ought to be, greater if the zero of the vernier falls without the scale, less if it falls within. Such faults often occur, and they produce what are known as *constant errors*, so called to distinguish them from accidental errors such as might arise from parallax (§ 13), or from mistakes in reading the scale, or from bad adjustment of the jaws to the ends of the object. It will easily be seen that they are very dangerous. We have pointed out (§ 17) that *accidental errors* may often be detected by making a number of independent observations and comparing the results, and that they may be allowed for, more or less, by taking the mean of several measurements. But *constant errors*, such as the one here supposed, would evidently not give rise to irregularities by which they might be detected, nor would the finding of the mean serve to get rid of them since they *always occur in the same direction*, and so do not balance one another in the long run. *Hence it is most important to guard as well as possible against constant errors.*

In the case before us, and in other similar cases, this can very easily be done. The instrument we are going to use *must evidently be adjusted so that its scale ought to give the reading zero, and if it does not do so we must observe what the reading is.* That reading measures for us what is called the **error of the instrument**. The first thing to be done then in beginning to use an instrument is to find out its error. Suppose now we have to find the error of our slide callipers. They are adjusted as already described, and then the position of the vernier zero is accurately noticed. Three cases may occur—

Case i. The zero of the vernier may coincide exactly with the zero of the main scale. In this case the *error is zero*, the callipers give true readings (assuming, as we must for the present, that the scales upon them are accurately marked).

Case ii. The zero of the vernier may fall inside the zero of the main scale—that is, between the graduations 0 and 1. In this case there is an *error*, and its effect is to make the readings *too large* by as much as the vernier zero is beyond that of the scale. This is called a *positive error.* If the first vernier graduation coincides with one of those on the main scale the *error is* + 0.1 *mm.*, which is as much as to say that the readings are *too large* by 0.1 *mm.*

Case iii. The zero of the vernier may fall outside the zero of the main scale. In this case too there is an *error*, but its effect is, this time, to make the readings *too small.* This is called a *negative error.* Suppose it were the eighth *vernier* graduation which coincided with one on the main scale. Were there no error coincidence would occur at the tenth. The *error* is thus − 0.2 *mm.*, which means that the readings are *too small* by 0.2 *mm.*

It will now be plain that, if there be an error, a **correction** will have to be applied to the reading. If the error is positive, the reading is too large and the amount of the error must be subtracted from it. If the error is negative, the reading is too small, and the amount of the error must be added to it. Hence the *correction* is obtained by *changing the sign of the error* and then *adding* to the readings. Thus to correct for the *error of* + 0.1 *mm.* (Case ii. above) we *add the correction* − 0.1 *mm.* to all the readings, and for the *error of* − 0.2 *mm.* (Case iii.) we *add the correction* + 0.2 *mm.*

The *correction* required for every instrument used must henceforth be found by determining its *error* as described. This need not be done every time the instrument is to be used; it may be done at the outset, and put on record in some place where it is easy to find and refer to it. Occasionally the *error* must be redetermined to see whether it has altered, as it may if the instrument be used a great deal or be roughly handled, and, if necessary, the correction must be re-entered. This correction must *always* be added (paying attention to its *sign*) to every reading obtained with the instrument.

§ 30. USE OF SLIDE CALLIPERS.—The correction for the slide callipers having been found, it will now be possible to measure small distances more accurately than we have done hitherto.

Exercise X.

To measure the extension of various objects correctly to 0.1 *mm.*

Apparatus.—Slide callipers.
Objects to be measured. Small metal cube and cylinder glass scale of Exercise II.

Method.—Open callipers, insert object, slide movable jaw up till *nearly* touching, clamp, and finish with fine adjustment. Read scale and vernier and enter reading (uncorrected). Release, and open jaws again. Make several measurements of each length and enter each result separately. Then *add the correction* (mind its *sign*) to each of these, and take the mean of the corrected results.

Measure in this way the length, breadth, or thickness of the cube and the diameter of the cylinder. Also, find the width of the glass scale and compare the mean corrected result with that found by use of common callipers in Exercise VII. (§ 21). This will give an idea of the correctness of your *estimations* in that Exercise.

§ 31. THE WEDGE.—We saw, in § 22, that if a straight line be made to slope or advance by a certain amount in crossing a certain space, then, by dividing that space into equal strips, points can be marked off at which the line has advanced a definite fraction of that amount. Thus, if a line advanced 0.5 *cm.* in crossing 50 *cm.*, and this space were divided into 50 strips by lines 1 *cm.* apart, these would mark points where the sloping line had advanced $\frac{1}{50}$ of 0.5, $\frac{2}{50}$ of 0.5, . . . $\frac{49}{50}$ of 0.5 *cm.* Thus we could measure to 0.01 *cm.* The whole distance which the sloping line advances may be called *the pitch.* In the case supposed the *pitch* is 0.5 *cm.* It will be plain that *the smaller the pitch*, and *the larger the number of strips* into which the space crossed is divided, the smaller will be the distances marked off. It has also been said that the disadvantage of a linear scale made on this plan lies in the difficulty of *seeing* very small distances. But a distance too small to be seen accurately is easily *felt*, and if the principle of the diagonal scale could be *applied to end measurement* we might hope to measure distances still smaller than those we have attempted by line measurement (in Exercises III. to VI.), or by combined line and end measurement (in Exercises VII. to X.). Before describing the instrument in which this is done most perfectly, we will construct for ourselves one, called a **wedge**, in which it is done very simply, and which is extremely useful for making measurements that cannot easily be made with ordinary scales, such as that of the inside diameter of a tube.

Exercise XI.

To make a wedge capable of measuring to 0.05 *cm.*

Apparatus.—Thin cardboard, ruler, drawing-pen, compasses, steel scale, sharp penknife, set-square.

Method.—Draw a straight line, *ab*, on the cardboard exactly 10 *cms.* long. At each end draw perpendiculars, *ac* and *bd*, exactly 1 *cm.* long. Join *cd*, and draw the diagonal *ad.* Then divide *ab* and *de* into 20 equal parts by marking off with the linear protractor (as in Exercise I. § 10) points 0.5 *cm.* apart. Join each of these to the one immediately above or below. Then *very carefully* cut out the wedge, *adb* (*Fig.* 15).

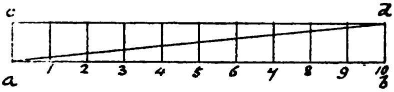

FIG. 15.

The edge, *ad*, evidently rises 1 *cm.* between *a* and *d*, and where it is cut by each of the vertical lines it has risen $\frac{1}{20}$ of this, or 0.05 *cm.* Number the graduations, beginning from the point, the *second* being marked 1, the *fourth* 2, and so on. Evidently these numbers represent the width (in *mms.*) at each graduation. The wedge is seen to have a "pitch" of 1 *cm.*

Exercise XII.

To measure the inside diameter of a glass tube.

Apparatus.—Wedge made and graduated in Exercise XI.
Two or three pieces of glass tubing of various sizes.

Method.—The piece of tubing is held vertically, and the wedge *gently* lowered into it till it *just* rests upon the edges. Evidently if one of the graduations happens to fall upon the edges the diameter may be read off directly. If not, read off the number of *mms.* and half *mms.* and then estimate the *number of fifths of a graduation*, and add.

Thus : wedge falls below 5th graduation ∴ *diameter* = 0.25 + ...*cms.*
Estimated as ⅘ beyond ,, ,,
Each ⅕ graduation = 0.01 *cm.* ∴ *diameter* = 0.29 *cms.*

The wedge might evidently be made more accurate by diminishing the "pitch" or by increasing the number of graduations, or both. Wedges of thin metal may be bought having a pitch of 1 *cm.* and divided into 100 strips, thus enabling us to read directly to 0.01 *cm.*

§ 32. THE SCREW.—To understand how the principle just used may be applied still more successfully, we proceed as follows :—

Exercise XIII.

To draw a diagonal scale and show how it may be converted into a diagram of a screw.

Apparatus.—Ruler, steel scale, compasses, set-square, drawing-pen, paper, wooden cylinder, about 3 *cms.* (not more) in diameter.

Method.—Draw, with pencil, a line exactly 10 *cms.* long, and at each end erect a perpendicular exactly 6 *cms.* long. Mark off, on these, points exactly 1 *cm.* apart, and join so as to form six strips, each 10 *cms.* long by 1 *cm.* wide. Then draw, with the pen, a diagonal of one strip and the parallel diagonal of each of the others. Each diagonal then advances exactly 1 *cm.* in passing from corner to corner, and we have formed six diagonal scales of pitch 1 *cm.* Now take the wooden cylinder and paste the paper, face outwards, carefully round it, taking care to make the horizontal strips meet exactly. It will be seen at once that *the diagonals now make one continuous line* running round the cylinder in a spiral, so that each point is exactly 1 *cm.* from the points vertically above and below it. This is a diagram of *a screw whose pitch is* 1 *cm.* In making *one complete turn* the line *advances* 1 *cm.* In two turns it advances 2 *cms.*, and in any fraction of a turn it advances that fraction of 1 *cm.* So in

$2\frac{1}{25}$ turns it advances 2.04 *cms.*, and so on. If, now, we imagine this line to project outwards from the surface of the cylinder it becomes *the thread* of our screw; and, lastly, if this thread were made to run in a suitable *fixed* groove, then on turning the screw the thread, in advancing, would draw along the entire screw with it. *One complete turn would cause the end of the screw to advance a distance of* 1 *cm.*, that is to say, a distance *equal to the pitch.* Similarly, the hundredth of a turn would advance the end through a distance equal to one hundredth of the pitch, and so on. By dividing the circumference at one end into any desired number of equal parts, and having a fixed pointer close to its edge, we could evidently measure any desired fraction of a turn.

If, then, we know the pitch of a screw, and the number of turns and fraction of a turn needed to bring the end of it from one point to another, we can at once tell the distance between these points. This plan is used in some instruments now to be described.

§ 33. THE SCREW GAUGE OR MICROMETER.—This is depicted in *Fig.* 16. It consists of a strong U-shaped piece of metal *abc*, inside the end *a* of which a small steel stud *d* is fixed, while to the end *c* is attached a long collar *cf.* In this is a grooved hole through which works a long screw *s*, having at one end a steel stud *g*, like the one at *d*, and at the other end a milled head *h*, which has attached to it a sheath *k* that fits over the collar *cf.* The lower edge of this sheath is bevelled, and is divided into a certain number of equal parts. Along the collar *cf* runs a horizontal line with a scale having divisions equal to the pitch of the screw. When the studs *d* and *g* are made just to touch, the graduation of this bevelled edge marked ← should coincide with the zero of the fixed scale. In the usual form the pitch of the screw is 0.5 *mm.*, and the *bevelled edge* carries 50 equidistant marks. Five of these, corresponding each to $\frac{1}{5}$ of a whole turn, are large and evidently register *tenths* of a *mm.* Each of these large divisions is further subdivided into 10 parts, so that the smaller marks register *hundredths* of a *mm.*

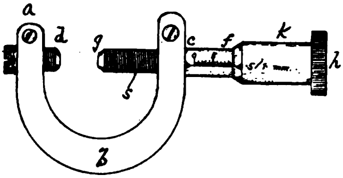

FIG. 16.

It will be noticed that each of the larger graduations is marked with *two* numbers, thus—.5/1, .4/.9, .3/.8, .2/.7, .1/.6. The reason for this is easily understood. The alternate marks on the *fixed scale*, purposely made more prominent, correspond to *whole mms.* It is convenient to read off the *tenths* directly from the larger marks on the edge. Now of these the one marked ← will evidently always fall upon one of the scale divisions whenever it comes to the line. It is therefore (except at the zero) upon either a *whole mm.* or a *half mm.* mark, and so is numbered .5/1. The *next* large mark which comes round *as the screw is turned back* corresponds to a distance 0.1 *mm. greater* than that of the last scale division passed. If the edge has just passed a *whole mm.* mark it is read .1, but if the edge has just passed a *half mm.* mark it must clearly be read .6. Hence it is numbered .1/.6, and so with the rest.

The following will then be the method of reading the instrument:—

(i.) Count the *whole mms.* exposed on the horizontal scale.

(ii.) Notice whether the bevelled edge has last passed (*a*) a whole *mm.* mark or (*b*) a half *mm.* mark. Then read off the first decimal place from the nearest large division *below* the line, taking the smaller number in case (*a*), the larger in case (*b*).

(iii.) Find the second decimal place by counting the number of small divisions between the large mark just read and the horizontal line.

§ 34. USE OF MICROMETER.—The error of the instrument (§ 29) must first be found. To do this the milled head is turned back, so that the studs are a little way apart, and then grasped *very lightly* between the thumb and finger and turned forward till the studs just meet. When this happens the thumb and finger will, if the grasp be light enough, *slip* on the milled head. (It is most important not to grip hard and screw up tightly, as this will strain the instrument and rapidly produce a serious error.) The graduation of the edge marked ← should now coincide with the zero of the scale. If it falls above the line the error is negative and the correction therefore positive; if it falls below the error is positive and the correction negative. In the best forms of the instrument the stud *d* can be adjusted if necessary by means of a small screw, and the error is always kept at zero.

Exercise XIV.

To measure the thickness of various objects correctly to 0.01 *mm.*

Apparatus.—Screw gauge, wooden clamp.
Pieces of wire, microscope cover-glass.

Method.—The micrometer is fixed in the clamp, being gripped at *b* (*Fig.* 16) so that *cfh* is horizontal. The object is held loosely in the left hand between the studs and the milled head turned (cautiously, as described above) till the rotation stops of its own accord. The reading is then taken. Several independent measurements are made at different parts of the object. Find the diameter of one or more pieces of wire, and the thickness of the cover-glass. Give the *mean corrected results* (see Exercise X., § 30).

§ 35. THE SPHEROMETER.—In this instrument, depicted in *Fig.* 17, the principle of the screw is used to get still finer measurements. It consists of a strong metal tripod, the legs of which are exactly equal in length and are so placed as to fall at the corners of an equilateral triangle. Through a collar in the centre works a fine screw *s*, having a milled head *h*, and carrying a large metal disc *d*, the circumference of which bears a number of equidistant marks. At the lower end the screw has a rounded point. Fixed to one of the legs, so that its inner edge is close to the edge of the graduated disc, is a vertical strip of metal *c*, upon which is marked a scale whose divisions are equal to the pitch of the screw. In the common form the pitch of the screw is 0.5 *mm.*, and the disc has 250 divisions. There are fifty large marks,

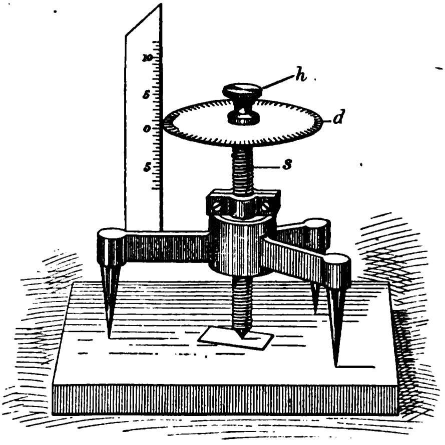

FIG. 17.

each of which thus corresponds to a movement of the end of the screw of 0.01 *mm.*, and each of these divisions carries five smaller marks, every one of which evidently corresponds to a movement of 0.002 *mm.* In the more expensive forms there are 500 divisions, so that measurements can be made accurate to 0.001 *mm.* There is also supplied with the instrument a glass plane, specially ground, perfectly flat, upon which it stands when in use. The principle is evidently precisely like that of the screw-gauge. The plane surface corresponds to the fixed stud, the divided disc to the graduated sheath, and the vertical scale to the horizontal scale.

§ 36. USE OF THE SPHEROMETER.—The error must first be found. The instrument is placed upon the glass plane, and the milled head is turned (with precautions mentioned in § 34) so that the screw is lowered till, on touching the top lightly with the finger, a slight rocking can be detected by a tapping *noise* due to contacts of the legs with the glass plate. The screw is then turned back through one large division of the divided circle and the instrument lightly touched again. This is repeated, if necessary, till a large division is found where no rocking can be heard. Then the screw is moved forward through half a large division (estimated). Suppose now the instrument rocks again. The point sought is between the half and whole division. The screw is then moved back through one small division at a time till it just fails to rock when touched. The zero of the vertical scale ought now to have the mark ← of divided circle exactly opposite to it. If not, the error is read off and the required correction put on record.

Exercise XV.

To measure a thickness correctly to 0.002 *mm.*

Apparatus.—Spherometer, microscope cover-glass used in Exercise XIV.

Method.—The cover-glass and the glass plane are carefully dusted. The screw is raised and the cover-glass laid below the end of it. Then the screw is lowered and the precise point when it touches the cover-glass determined in exactly the way described above. The reading of the scale and circle gives thickness of cover-glass. Several measurements should be made at different parts, and the mean corrected result compared with that of Exercise XIV.

§ 37. OTHER USES OF THE SCREW.—We have seen that, by taking advantage of the property possessed by a uniform screw of moving forward a definite amount when turned round in a grooved collar, it is possible to measure very small distances indeed. This property is used in a most important instrument, called a *Dividing Engine*, for the construction of finely graduated scales. A long and very uniform screw of small pitch is furnished with a large divided head, the revolutions of which (like those of the instruments just described) can be counted by means of a linear scale fixed alongside it. The other end of this screw is made practically to carry an instrument, the principle of which is the same as that of the Linear Protractor. In fact, instead of a steel point having to be lifted backwards and forwards (as in our Exercises I. and II.) the marking point is moved forward to any desired extent by turning the screw. In this way scales of any required degree of fineness can be made. For example, the vernier upon your slide callipers, the divisions of which are 0.9 *mm.* long, was graduated in this way. We shall see a few other applications in the course of our work.

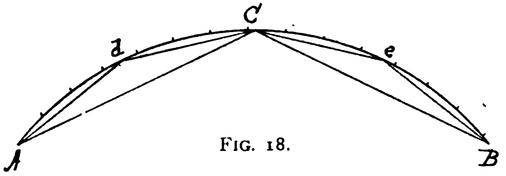

FIG. 18.

§ 38. LENGTH OF A CURVED LINE.—So far we have been measuring the lengths of straight lines only. We have now to contrive means for measuring those of curved lines. Consider a simple case such as that of *Fig.* 18. The points *A* and *B* are joined by the curved line *ACB*. If one of our *cm.* scales be laid upon the figure it is plain that the distance from *A* to *B* by way of *ACB* is longer than the straight line *AB*. If we go out of our way to touch the point *C* we shall find a distance equal to the sum of the straight lines *AC* and *CB*, which is greater than *AB*, but, since *AC* and *CB* are themselves "short cuts," is still less than the curved line. Were we to touch at the intermediate points *d* and *e* we should get still nearer to the truth, and by touching at a sufficient number of points we could measure a large number of short straight lines whose total length would be *almost* equal to that of the curved line.

Now this can be done practically by the use of the *Spring*

Compasses, which are made so that they open of their own accord and are closed by turning a nut upon a small screw. This provides a fine adjustment by which the needle-points can be set accurately any desired distance apart (*Fig.* 19).

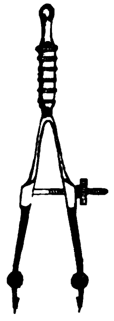

FIG. 19.

Exercise XVI.

To measure the length of a curved line.

Apparatus.—Spring compasses, *mm.* scale, paper marked with curved line (as in *Fig.* 18).

Method.—The points of the compasses are set exactly 0.5 *cm.* apart. One of them is placed on *a*, the other on the nearest point of the curve, and, keeping the further point down and raising the nearer, the compasses are turned over and over so as to follow the course of the curve. Count carefully the number of turns required to pass from *a* to *b*. Most likely the curved line will not contain an exact number of half *cms.* If this be so, find the length of any little remainder between the last half *cm.* mark and *b* separately with the compasses, transfer to the scale, and add to the length already found. Make at least two independent counts and give mean result.

§ 39. APPROXIMATION.—The measurement just made is evidently not accurate. Every one of the *half-cm.* steps was a "short-cut." It would have been more accurate had the points of the compasses been set closer together, but this makes the experiment harder to do. A result that we know is not accurate but only as nearly so as we can conveniently make it is called an *approximate* result, or, shortly, an **approximation**. From what we have done already it will be clear that *all* our measurements are really approximations, for none of them are perfectly accurate, but we keep the name approximation for those which we *know to be a long way from the truth*, but which we cannot make better with the means at our disposal.

§ 40. LENGTH OF A CURVED LINE (*contd.*).—We can, then, obtain a much better approximation to the length of a curved line by making the number of points at which the curve is touched in passing from one end to the other very large. This can be done with a piece of ordinary glazed sewing cotton.

Exercise XVII.

To re-determine the length of the curved line of Exercise XVI.

Apparatus.—Paper marked with curved line, glazed sewing cotton, *mm.* scale, sharp penknife.

Method. — Cut one end of the cotton off clean, and lay it upon *a*, holding it there with the left forefinger. Then lay the cotton carefully *along* the curve for a *little way*, pressing it down with the right forefinger and, holding it in position, bring up the left forefinger as near as possible. Continue thus till the cotton has been laid along the curve right up to *b*, and at *b* cut it off with a sharp knife. The distance between the ends of this piece of cotton will be very nearly equal to the length of the curved line and can be measured by laying the cotton quite *straight* along the *mm.* scale. Compare the results with those of Exercise XVI.

In all cases a curved line *is considered for purposes of measurement as made up of a very large number of short straight lines*, and thus our measurements of curved lines are always approximations, but with care they may be made fairly near the truth.

§ 41. SUMMARY AND ADDITIONS. — We have learned from our experiments so far that in order to discover the truth about any object it is not enough merely to look at it or to touch it. The senses are easily imposed upon, and the reports which they give us often need to be set right. This, we have seen, may be done in several ways. We may arrange the conditions under which the senses are used so as to avoid making mistakes (*e.g.* § 13); we may recognise accidental errors by taking *several* (§ 17) *independent* (§ 18) *measurements* and *comparing the results*, and we may reduce the effect of these accidental errors by the device of taking an average, or better still, a *mean* (§ 17). We have seen, moreover, that there are other errors (§ 29) due, not to mistakes in observation, but to faults in the instruments used, and have noticed that these *constant errors* are specially dangerous, but that they may be avoided by applying *corrections* to our results.

Further, we have seen that even when all these precautions are taken, our measurements are not quite exact. We began by making mere rough comparisons with coarse scales and by the sense of sight alone, trusting to *estimation* (§ 15) for the finer part of our measurement (Exercises III.-VI.). The results were accurate only as far as the *whole mms.* Next we saw how, by applying the principle of the *vernier* (§§ 23, 24), our measurements might be made more delicate, and by combining, in the *slide callipers*, end-measurement with the use of a vernier were able to make our measurements, when duly corrected, accurate to *one-tenth part of a mm.* And finally, we learned how, by means of the principle embodied in the *diagonal scale* (§ 22), the *wedge* (§ 31), and, above all, the *screw* (§ 32), we might attain still greater accuracy, measuring with certainty to the *hundredth of a mm.* with the micrometer and to the *five-hundredth* (or even thousandth) *of a mm.* with the spherometer. It is evident that we have thus made our measurements more and more nearly true, but that—since there is no reason (save expense) why we should not carry them to the ten-thousandth of a *mm.*, and so on, by making finer screws with larger heads—none of them is really true. When we came to deal with curved lines we saw that we are obliged to be content with far rougher measurements or *approximations.* Therefore we see that while none of our measurements is absolutely accurate, some of them are more accurate than others. But, as we shall see, the object of making these measurements is largely that they may be compared among themselves. What, then, is the use of making one measurement accurate to the hundredth part of a *mm.* when another, with which we have to compare it, may be as much as a whole *mm.* out? The answer is not hard to find. Suppose we have to measure two lengths, one of 12.50 *cms.*, the other of 0.25 *cm.*, and that we make exactly the same mistake, namely, measure the length 0.05 *cm.* too small, in each case. The lengths will then appear as 12.45 *cms.* and 0.20 *cm.* respectively. It will be seen at once what a difference this makes to the relation between them. If we divide the greater by the less the true values give us 12.50/0.25 = 50, while our values would make 12.45/0.20 = 62.25, a number greater than the true one by nearly a quarter of the whole. Nor is it difficult to understand how this comes about. The error of 0.05 *cm.* is only $\frac{1}{250}$ of the larger length, but it is $\frac{1}{5}$ of the smaller. Had we been only $\frac{1}{250}$ out with the smaller, our error would have been in that case only 0.001 *cm.*, and we should have estimated our two lengths as 12.45 *cms.* and 0.249 *cm.* Now 12.45/0.249 = 50, and we see that when the two errors, instead of being equal, bear the same relation to the distance measured, then the *relation* between the two results remains unchanged. What is important, then, is the proportion which our error bears to the distance we are measuring, and we do not content ourselves with considering merely the whole error. It is usual to reckon what the error would have been if the distance measured had been 100 units. Thus in the case supposed above the error in the case of the larger length was 0.05 units in 12.50 units, so that in 100 units it would have been 0.4 unit, while with the shorter, being 0.05 in

0.25 units it would, in 100 units, have been 20 units. This method of reckoning *percentage error* is of the greatest importance. Very often in this book the true results have been or will be stated for comparison with the results obtained by the student. In all such cases the error of the experiments must be stated as a percentage error. It is of course in measuring small distances that the percentage error is likely to be largest, and, in fact, it is usually not easy to bring it below 0.1 per cent, but *in no case must a greater error than one-half per cent be allowed.* It is only because of the difficulty of dealing with very short distances that so large an error as this is permitted. Where all the distances to be measured are fairly large the error ought to be *much less than* 0.5 *per cent.*

For the present an exception may be made in the case of curved lines where, as we saw (§§ 38-40), we have to be content with rough approximations, and our error may amount to as much as 1 per cent, but we shall learn later on to avoid this in particular cases.

CHAPTER IV

AREA

§ 42. AREA.—We have now seen how to measure the length, breadth, and thickness of an object. These three names all represent the same kind of magnitude, namely, extension in one direction in space, and differ only as regards the direction in which that extension is measured (§ 4). We may now go on to consider that property of objects which depends upon their having *both* length and breadth. This is the amount of surface that they cover, or, in short, their *area.*

To measure area we shall need a *unit of area*, and, since the extent of a surface depends upon its length and breadth, we shall choose one that depends upon our unit of length. *The square, each of whose sides is equal to the unit of length, is said to have unit area.*

Thus if we use the British unit, we should take a square each of whose sides is 1 yard long, and our unit of area would be the *square yard.* In the metric system we should have the *square metre.* Of course we may use any of the multiples or divisions of the unit of length and have such quantities as a *square inch* or a **square centimetre.** In the following exercises we shall nearly always use the *sq. cm.* and sometimes, for very small areas, the *sq. mm.*

§ 43. FUNDAMENTAL AND DERIVED UNITS.—It should be noticed that there is an important difference between the unit of area and the unit of length. For the unit of length we are perfectly free to choose any distance we please. Reasons very plain to be seen lead us, in practice, to choose that distance which has been almost universally agreed upon, but we are not bound to do so. In short, the unit of length depends upon nothing but the choice of its user. It is called a **fundamental unit.** But it is otherwise with the unit of area. What this will be depends upon what the unit of length is. Hence, when we have once chosen our unit of length, we are no longer free to choose any unit of area we like—our previous choice decides its size. The unit of area, then, is said to be derived from the unit of length, and is called a **derived unit.** The caution given in § 9 must be recalled here. Not only is it important to state the unit of length we are using, but in dealing with areas it is most important to remember that we are dealing with a unit *derived from* that of length and to write the word "*square*" or the abbreviation "*sq.*" before the name of that unit.

§ 44. AREA OF A RECTANGLE.—We will first find by experiment the area covered by a rectangle whose length and breadth are known.

Exercise XVIII.

To determine the area of a rectangle.

Apparatus.—Paper, set-square, ruler, steel *mm.* scale.
Drawing-pen, compasses, sharp penknife.

Method.—Draw a straight line *AB* exactly 6 *cms.* long and at each end draw perpendiculars, *AC* and *BD*, in the same direction, each 4 *cms.* long. Join *CD*. Then the figure *ABCD* is a rectangle. Now divide *AB* and *CD* each into six equal parts and draw the vertical lines joining these points. The rectangle is thus divided into six equal strips each of which is 1 *cm.* wide. Therefore the area of *ABCD* is six times that of one strip (*Fig.* 20). Next find the area of one strip by dividing each of its sides into four equal parts and joining so as to cut the strip into four small rectangles each 1 *cm.* long and 1 *cm.* wide. It is evident that each of these is *a square centimetre*, and that the area of one strip is equal to 4 *sq. cms.*

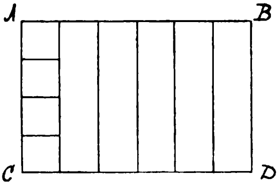

FIG. 20.

Therefore the area of the rectangle is 6 × 4 *sq. cms.* = 24 *sq. cms.* By similar means we can find the area of any other rectangle and so arrive at a *general rule* for finding the area of a rectangle.

Measure the length in any unit (e.g. *cms.*); measure the breadth *in the same unit;* multiply the number expressing the length by the number expressing the breadth, and the product is the number which gives the area in *square* units (*cms.*, etc.).

§ 45. MULTIPLES AND SUBDIVISIONS OF THE UNITS OF AREA.—Since a square is only a rectangle whose breadth is equal to its length, the student will now have no difficulty in *proving* the following relations, which must be carefully borne in mind :—

1 SQ. YARD = 9 *sq. ft.* = 1296 *sq. ins.*
1 *sq. ft.* = 144 *sq. ins.*
1 SQ. METRE = 10^2 *sq. decimetres* = 10^4 *sq. cms.* = 10^6 *sq. mms.*
1 *sq. decimetre* = 10^2 *sq. cms.* = 10^4 *sq. mms.*
1 *sq. cm.* = 10^2 *sq. mms.*

and in deducing any others required.

He may also determine the *relation between the British and the Metric units of area* from the numbers obtained in Exercise V. (§ 16) and compare the results with those given in the following table :—

1 SQUARE YARD = 0.836 *sq. met.*
1 *square foot* = 929.03 *sq. cms.*
1 *square inch* = 6.45 *sq. cms.*
1 SQUARE METRE = 1.196 *sq. yd.*
1 *square decimetre* = 0.1076 *sq. ft.*
1 *sq. centimetre* = 0.155 *sq. in.*

It is quite likely that the comparison will reveal serious differences, and we will now show why this may happen.

§ 46. MULTIPLICATION OF ERRORS.—We have seen that an area is measured by the product of two lengths, and before going further must notice carefully a very important matter. In measuring lengths, for example, with the glass scale there is no difficulty in getting the number of whole *cms.* and of tenths (first place of decimals) quite right, but the number of hundredths (second place of decimals) has to be *estimated.* Now practice and care are needed to make this estimation accurately within 0.01 *cm.* Thus suppose we have two lengths whose *true* values are 5.427 and 4.636 *cms.* One observer might estimate them as 5.43 and 4.64 *cms.* and another as 5.42 and 4.63 *cms.*, and *yet neither observer would make a mistake of as much as* 1 *unit in the second place of decimals.* Suppose now we required the product of these. For the first observer we have 5.43 × 4.64 = 25.1952 *sq. cms.*, and for the second 5.42 × 4.63 = 25.0946 *sq. cms.* The *true* value would of course be 25.1795 . . . *sq. cms.* Thus, although the error in measuring each length was *less than* the tenth part of a *mm.*, the values found for the area actually differ by more than 0.1 *sq. cm.* (that is, by more than 10 *sq. mms.*). This teaches us two things : (i.) That great care must be taken in measuring the lengths from which areas are to be calculated. [It is evident that, whenever possible, we should use the slide callipers with their vernier so as to make as sure as we can of the second place of decimals.] (ii.) That it is utterly useless and misleading to give areas to four places of decimals when there is any doubt about the second decimal places in the lengths. Even the first of the four is doubtful when we have to rely on estimation entirely. In no case, *unless* our lengths have *both* been measured with the micrometer, so that we are perfectly sure of the second decimal place in them, is it any use to give the area to more than two places of decimals. In the following exercises therefore the student should, after multiplying the lengths together, strike off all decimal places after the second, *or if he has nothing but estimation to go upon*, all after the first. In doing this it must be remembered that in striking off figures from four places, for example, the number removed represents so many ten-thousandths of 1 *sq. cm.* It is removed, *not because it is known to be wrong, but because its value is uncertain.* We wish, in striking it off, to *alter the result as little as possible.* Hence if we are retaining two places we shall simply strike off the last two digits if they are 50 or less, and shall strike them off and add 1 to the second figure retained if they are more than 50. Similarly, if we are keeping only one place we shall add 1 to the figure kept if the three rejected digits form a number over 500.

Applying this to the case above, the first result, 25.1952 becomes 25.2, and the second, 25.0946 becomes 25.1. [The true value is, as we have seen, very nearly 25.18.]

§ 47. AREA OF RIGHT-ANGLED TRIANGLE.—In most cases the areas we have to measure are not rectangles. We may have other quadrilateral figures, or triangles or polygons, and we have to inquire how our method of measuring areas applies to these other rectilinear figures.

We will consider first the case of *a right-angled triangle.*

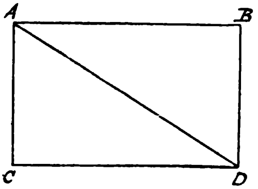

FIG. 21.

Exercise XIX.

Apparatus.—Same as for Exercise XVIII.

Method.—Draw a rectangle *ABCD* as in the last exercise. Join *AD* (*Fig.* 21). Then *AD* is the diagonal and divides the rectangle into two equal parts. *Prove* this by cutting carefully along *AB*, *BD*, and *DA*, and placing the triangle thus cut out upon the triangle *ACD*, so that *B* falls upon *C*, etc. If the two coincide exactly they are equal in area. [Notice carefully the difference between this *experimental* proof and the one given by *Euclid* (I. 34), and notice too that the method of superposition is that carried out in imagination in the 4th Proposition of *Euclid's* First Book.] We have now arrived at a method of finding the *area of a right-angled triangle.* It is exactly *half that of a rectangle whose length and breadth are the lengths of the sides containing the right angle.* But we know, from Exercise XVIII., what is the area of a rectangle whose length is l units and breadth b units; it is $l \times b$ *square units.* So with a right-angled triangle, if the sides containing the right angle be l units and b units long respectively, the area is $\frac{l \times b}{2}$ square units.

§ 48. AREA OF ANY TRIANGLE.—If we have to deal with a triangle that does not contain a right angle we may have two

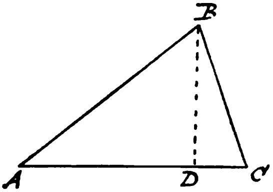

FIG. 22.

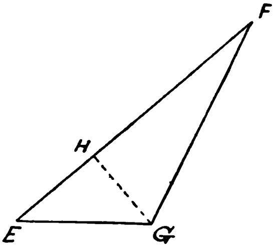

FIG. 23.

cases. The triangle may be acute-angled, as ABC (*Fig.* 22), or obtuse-angled, as EFG (*Fig.* 23). By drawing BD perpendicular to AC and GH perpendicular to EF respectively, each of them *can be divided into two right-angled triangles*, the areas of which are found as in § 47. Thus—

$$\text{area of } ABC = \text{area of } ADB + \text{area of } CDB = \tfrac{1}{2}(AD \times DB) + \tfrac{1}{2}(CD \times DB)$$
$$= \tfrac{1}{2}DB(AD + CD) = \tfrac{1}{2}(DB \times AC).$$

Similarly, the area of $EFG = \frac{1}{2}(GH \times EF)$.

Hence the *area of any triangle* is *one half the product of the length of the perpendicular from one angle on to the opposite side and the length of that side.* The side on to which the perpendicular is let fall is often called the *base* and the above rule expressed shortly as *area of a triangle* = ½(*base* × *perpendicular height*).

§ 49. APPLICATION.—We have now to apply this rule and inquire whether it is *general.*

Exercise XX.

To measure the areas of two triangles.

Apparatus.—Paper, ruler, drawing-pen, set-square, glass scale.

Method.—Draw two triangles *ABC* (acute-angled) and *EFG* (obtuse-angled). Draw a perpendicular from *A* on to *BC*. Measure *very carefully* the lengths of this and of *BC*, and from the mean values calculate the area of the triangle (*remembering the caution of* § 46). If the lengths are in *cms.*, in what unit is this area expressed?

Then draw perpendiculars from *B* on to *AC* and from *C* on to *AB* and obtain two other independent measurements of the area of *ABC*. Are all three results the same? What does this teach you?

Turning now to the triangle *EFG* we notice that there is only one angle (*G*), from which a perpendicular can be drawn in the ordinary way to the opposite side (*EF*). Draw it, and measure the area of *EFG* as above. If now we try perpendiculars from *E* or from *F* on to *FG* or *EG*, we find that these sides must be produced if they are to be cut by the perpendiculars. Let the perpendicular from *E* on to *FG* cut *FG* produced at *K*. Measure the lengths of *EK* and *FG* and take $\frac{1}{2}(EH \times FG)$. Does this give the area as already found? Repeat measurement by drawing *FM* at right angles to *EG* produced. Does this result agree with the other two? What does this prove to you?

§ 50. OTHER RECTILINEAR FIGURES.—We have now proved by experiment that the area of a triangle is obtained by taking half the product of the length of one side and the vertical distance between the direction of that side and the opposite angle of the triangle. This is usually stated shortly as follows:—

area of a triangle = ½(*base* × *vertical height*).

Notice that this experimental result furnishes a proof of the theorem that "triangles upon equal bases and between the same parallels are equal" (*Euclid*, I. 38), and also involves the truth of Euclid's Propositions, I. 37, 39, and 40. But having the means of measuring the area of a triangle we can readily measure that of *any figure bounded by straight lines*, for it is always possible by joining opposite angular points to *divide such a figure into a number of triangles.* Thus any *quadrilateral figure* is divided by a diagonal into two triangles, while *polygons* may be divided into three or more. For example, the irregular hexagon of *Fig.* 24 is divided by the three dotted lines AC, CF, and FD into four triangles whose areas can be measured as in Exercise XX., and added together to find that of the hexagon.

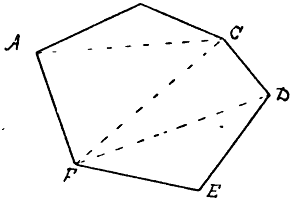

FIG. 24.

§ 51. FIGURES BOUNDED BY CURVED LINES.—So far we have dealt with rectilinear figures; we have now to consider figures bounded by curved lines. We saw in §§ 38-40 that a curved line must, for purposes of measurement, be considered as made up of a very large number of short straight lines. This idea will help us to understand how the area of a figure enclosed by a curved line can be measured, for if we think of that curved line as made up of a number of straight ones, we must also think of the figure bounded by it as a kind of polygon. We cannot, however, obtain more than a very rough approximation of the area by such a plan, for if we make our short-cuts small enough to be anywhere near the truth, we shall evidently get triangles much too small to be dealt with. But there is one simple case that we may treat successfully in this way. The figure called a *circle* is bounded by a curved line, but it is perfectly regular in shape, and hence if we try to think of the circle as a polygon, by imagining the circumference to consist of an enormous number of short straight lines, we can see that this imaginary polygon would be a *regular* one.

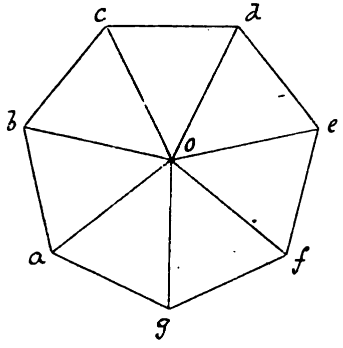

FIG. 25.

§ 52. AREA OF A REGULAR POLYGON.—We have seen, in § 50, how to measure the area of a polygon by dividing it into triangles. When the polygon is *regular* (has all its sides equal) we can simplify this process, for there is within a regular polygon a point called its centre, equally distant from each of the angular points, and by joining this to the angular points, the polygon is divided into as many equal triangles as it has sides. Take, for example, the regular heptagon *abcdefg* of *Fig.* 25. By drawing from the centre *o*, *oa*, *ob*, . . . *og*, it can be divided into seven equal triangles. Now each of these has an area equal to one half the product of the length of a side of the heptagon and the perpendicular distance from the centre to the side. Evidently the seven together have an area equal to one half the product of the length of all the sides of the heptagon, taken together, and the perpendicular distance from centre to side.

Similarly, for any *regular polygon*, we have—

$$area = \tfrac{1}{2}(perimeter \times perpendicular\ distance\ from\ centre\ to\ side).$$

§ 53. AREA OF A CIRCLE.—Let us now apply this to the case of a circle. Here the polygon is imagined to have so many equal sides that their length together amounts to the same as that of the *circumference*. Again, there are so many of these sides that each is extremely short, a mere point in fact. Hence the perpendicular distance from the centre to any one of them is simply the *radius* of the circle. Thus—

$$area\ of\ a\ circle = \tfrac{1}{2}(circumference \times radius).$$

To find the area of a circle, then, we must know the lengths of its circumference and of its radius. And here we meet with a difficulty. We have seen (§ 38, etc.) that we cannot measure the length of a curved line exactly, and it is not easy to say how near our approximation is. Observation, however, teaches us one thing. The size of a circle depends upon its radius, and the greater the radius the greater the circumference. If now we could find any regular relation between the radius and the circumference, by making a series of measurements once for all, we might be content afterwards to measure the radius, which is easily done, and calculate the length of the circumference. But is there any regular relation? That we have now to find out. Since it may happen, for example, in dealing with a penny, that we cannot readily find the centre of our circle in order to measure its radius, we will take the diameter and compare the length of it with the length of the circumference.

§ 54. DIAMETER AND CIRCUMFERENCE OF A CIRCLE.

Exercise XXI.

To seek a relation between the length of the circumference of a circle and that of its diameter.

Apparatus.—Paper, ruler, compasses (with pen), drawing-pen, steel scale, glazed sewing-cotton, glass scale, common callipers.

Also, circular piece of tin, about the size of a half-crown, having a radial line scratched upon one side.

Method.—Draw three circles, of different sizes, on the paper, and rule a diameter of each. Measure, with the glass scale, the length of the diameter of one circle (§ 14), and with a piece of glazed cotton measure *as accurately as possible* the length of its circumference (§ 40). Divide the length of the diameter into that of the circumference, carrying your work to 4 places of decimals. [To do this accurately work to 5 places, and then if the 5th digit be 5 or less than 5, simply strike it off, but if it be 6 or greater than 6, strike it off and add 1 to the 4th digit. Why?]

Now make similar measurements and calculations with each of the other two circles. Calculate the percentage differences (§ 41) between the numbers found. What do you notice about the ratio of circumference to diameter?

Next take the circular piece of tin. Draw a straight line across the paper, and place the disc upright on this line, near to one end, and so that the radial line upon it just touches, and is perpendicular to, the ruled line. Make a very small mark on the ruled line where this contact occurs, and then roll the disc round (taking care that it does not slip) along the ruled line till the radial line again touches it at right angles. Mark the point of contact as before, and with the glass scale measure the distance between these points. It is very nearly the circumference of the disc. Now measure the diameter of the disc with the callipers and glass scale, and as before calculate the ratio of circumference to diameter. Compare this number with the mean of the three found above. What do you think may be inferred from these observations?

§ 55. THE VALUE OF π.—The ratio, which we have just attempted to find, of the circumference of a circle to its diameter is a number of very great importance. It is, however, usually *represented*, not by a number, but by the Greek letter π. The reason is that its value cannot be expressed exactly by any number, for as we have seen, it is impossible to measure exactly the length of a curved line. Archimedes, who lived from 287 to 212 B.C., showed that its value is not far from $\frac{22}{7}$. [How does your mean result compare with this?] By calculations, too difficult to be described here, it has been shown that its actual value is *greater than* 3.1415926535 but *less than* 3.1415926536, and though the reckoning has been carried on to some hundreds of decimal places, it has been found that the decimal neither terminates nor recurs. It is, in fact, impossible to find a *straight* unit, however small, that will exactly divide a *curved* line, and hence the circumference and the diameter of a circle cannot both be measured exactly in the same unit. This is expressed by saying that they are *incommensurable.* We may, of course, by taking enough decimal places find a value of π which differs from the truth by as little as we please, but differ it always will. If we take *the value* $\frac{22}{7}$ and compare it with the best hitherto ascertained, we shall find that it *differs by less than* 0.05 %. This is an error well within the limit laid down as possible in § 41. If, as is often done, we take 3.1416, we shall have an error of only + 0.0023 %.

§ 56. CALCULATION OF THE AREA OF A CIRCLE.—Henceforth, since our means of measurement do not permit us to determine the length of the circumference of a circle with satisfactory accuracy, we shall be content to accept the value for the ratio π, which has been found by measurements more accurate and calculations more intricate than we can attempt as yet, and to *calculate* the circumferences of any circles we may have to deal with from measurements of their diameters. We have just seen that—

$$\text{circumference} = \pi \times \text{diameter},$$

and since the diameter is twice the radius, we have

$$\text{circumference} = \pi \times \text{radius} \times 2.$$

We have also seen in § 56 that

$$\textit{area of a circle} = \tfrac{1}{2}(\text{circumference} \times \text{radius}),$$

and

$$\therefore \textit{area of a circle} = \tfrac{1}{2}(\pi \times \text{radius} \times 2 \times \text{radius})$$
$$= \pi \times \text{radius} \times \text{radius, or } \pi \times \textit{radius}^2.$$

Thus if we measure the radius of a circle, and find its length to be r *cms.*, then the area of that circle is $\pi \times r \times r$ or πr^2 *sq. cms.*

Exercise XXII.

To measure the areas covered by circular discs.

Apparatus.—A fairly new penny and threepenny piece, glass scale, common callipers, slide callipers.

Method.—Find the diameter of the penny. Bear in mind that the diameter is the *greatest* distance across a circle (*Euclid*, III. 15). The measurement may be made with the common callipers. The penny and callipers are both held flat, and the jaws made just to grip the edges of the coin at its greatest width, so that on moving it never so little backwards or forwards they cease to touch. Transfer to scale and read (estimating to 0.1 *mm.*). As a check upon this, lay the glass scale upon the coin and read the greatest width between the edges. From the mean value of the diameter calculate the radius and thence the area. The area covered by the threepenny piece is found in a similar way, but its diameter being smaller must be measured still more carefully (why? § 40) and the slide callipers may be used.

Give the areas in *sq. cms.* What are they in *sq. mms.*? [Note again § 46.]

§ 57. AREA COVERED BY IRREGULAR FIGURE.—Seeing the difficulty we have had with even the most regular of figures bounded by a curved line it is hardly to be expected that the area of a plane surface, whose boundary is an irregular curved line, can be found practically by simple linear measurements. It will be shown later that there is no difficulty in measuring such areas *by indirect means.* It is, however, quite easy to get an approximate measurement by a process which practically consists in counting how many times a ruled *sq. mm.* can be laid upon the area in question. This is done by the use of so-called "*curve-paper*," which is ruled with horizontal and vertical lines 1 *mm.* apart, so that it is divided into *sq. mms.* Every tenth vertical and horizontal line is ruled more firmly, so that the whole *sq. cms.* (each containing 100 *sq. mms.*) are clearly marked.

Exercise XXIII.

To measure approximately the area of an irregular plane surface.

Apparatus.—Piece of thin cardboard or metal, not very large, and of irregular shape, curve paper.

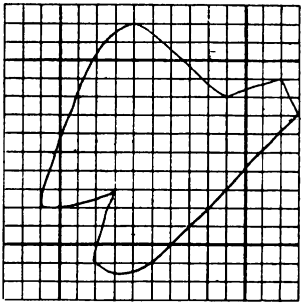

FIG. 26.

Method. — The object to be measured is laid upon the curve paper and a line drawn by running a very finely-pointed pencil round the edge of it (*Fig.* 26). This line may include a certain number of whole *sq. cms.* Number these. In addition there will be, round the edge of the figure, a number of fractions of *sq. cms.* Go round the edge counting the number of complete *sq. mms.* in each of these, numbering each as the count is completed, and finally add them up. Lastly, it will happen in some cases that the boundary line will pass right across a *sq. mm.* area. It is hard to estimate exactly fractions of *sq. mms.*, but a fair result may be obtained by noticing whether more or less than $\frac{1}{2}$ *sq. mm.* is included, counting as whole *sq. mms.* those portions that are certainly greater than half, and leaving out those that are obviously less. In *Fig.* 26 we have *no* whole *sq. units* + 74 *hundredths of a square unit* + 20 *hundredths of a square unit* from fractions, and the whole area is thus 0.94 *sq. unit.*

§ 58. AREA "COVERED BY" AND AREA "OF."—It must be noted carefully that the results which we have just obtained apply only to flat or plane surfaces. Thus in Exercise XXIII. (§ 59) we measured the area covered by a penny, but this was not the real area of the face or back of the coin. Owing to the impression stamped upon it the surface of a penny is irregular. If now we remember that a surface must be imagined as made up of lines lying side by side we realise that on the irregular surface of the penny some of these lines are curved. We have already seen (§ 38) that a curved line between two points is longer than the straight line between them, and hence, if a surface be made up wholly or in part of curved lines, we shall have to distinguish between the real area of that surface and the area covered by it. If a surface be not plane it *has* a greater area than it *covers.* For some purposes it is necessary to know the actual area of a curved surface. To do this we should evidently have to divide it into a large number of areas so small that each one of them is plane, and this *it is impossible to do practically*, just as it is impossible to divide a curved line into a number of straight ones. As with curved lines, so here, a simple solution has been found *by calculation* in certain cases those, namely, in which the lines forming the surface are regular curves. Of these we shall here mention only one.

§ 59. AREA OF SPHERE.—A surface every point of which is equally distant from one point, known as the centre, is called a sphere. A sphere, then, is the surface which a circle would mark out if we imagine it rotated about its diameter. It was first discovered by Archimedes, who gave the value $\frac{22}{7}$ for π (§ 55), that the area of such a surface is exactly four times as great as the area of the circle by the rotation of which it would be traced. Now the area of a circle whose radius is *r cms.* is πr^2 *sq. cms.* And hence

the area of a sphere whose radius is r cms. is $4\pi r^2$ *sq. cms.*

A method will be suggested presently by which this area can be found approximately, but *for the present* the student is asked to take the above result on trust (see §§ 74 and 144).

§ 60. WHOLE SURFACE OF OBJECTS.—It is easy to see how the whole surface of an object can be found provided its various faces are all bounded by straight lines. We have only to measure separately the area of each face (as in §§ 44 to 50) and add the results together. If any of the faces should be circular the area of such can be calculated from measurements of diameter (as in § 56), but in this case we meet with a difficulty, for an object with plane circular faces must evidently have some curved faces also. In the next chapter we shall describe two simple examples of such objects, and we will return to the question of their area there. The student should work the following problems before proceeding further:—

(6) Can you find any simple relation between the length of the side of a *cube* (Exercise X. § 30) and the whole area of its surface? If so, what is it?

(7) Which would have the greater surface, a rectangular block 4 *cms.* long by 3 *cms.* wide and 2 *cms.* thick, or a sphere 4.068 *cms.* in *diameter?* [Answer to 3 places of decimals only.]

(8) An object—called a tetrahedron—has four equal faces, each of which is an equilateral triangle. The length of one side is 6 *cms.* Find (by drawing and measurement) the whole area of its surface. What length of paper 3 *cms.* wide would be needed to cover it?

CHAPTER V

VOLUME

§ 61. VOLUME.—We have considered extension in one direction at a time (length, breadth, or thickness), and in two directions at once (area), and we come now to deal with extension in three directions at once. An object having extension in three directions is said to have a certain *volume*. Just as we have considered a surface as being formed by laying line upon line, and learned to recognise an area as the product of two lengths, so we have now to consider a solid object as being formed by laying surface upon surface, and to recognise a volume as the product of an area and a length, that is to say, of three lengths.

§ 62. PRISM.—A solid object which we can imagine to be formed by taking a large number of very thin *plane* figures (triangles, squares, octagons, etc.), all exactly alike in shape and size, and piling them up, with corresponding corners in straight lines, is called a **prism.** Thus by piling up a number of triangular laminæ (thin plates), we could make *a triangular prism.* In a similar way we could have a *hexagonal prism* (or pile of six-sided laminæ). Now such a pile might evidently be built up *either perpendicularly*, so that each of the straight lines formed by rows of corresponding corners is at right angles to the surface of the laminæ, *or obliquely*, so that these straight lines slope. Only the *former* case will concern us here. All the prisms we shall consider would be made by piling up the laminæ perpendicularly. They are called **right prisms.**

The right prism that would be made by piling square laminæ upon one another till the height was equal to the side of the square is called *a cube.*

The wooden blocks used in Exercise IX. (§ 26) were right prisms. If all the angles of a prism be right angles it is called a *rectangular prism*, and the cube is thus a special kind of rectangular prism. In other cases, we name the prism from the shape of the *base* (that is to say, of the laminæ which *might have been* piled up to make it). For example, we may have *a right isosceles triangular prism*, *a right rhombic prism*, and so on.

If the base is a circle, we do *not* speak of a right circular prism but of a *right cylinder*.

§ 63. UNIT.—Our unit of volume must evidently depend upon those of area and of length, and since the unit of area itself depends upon that of length, it is plain that the unit of volume is a *derived unit* (§ 43) depending on the unit of length.

We shall obtain it by taking the square whose side is equal to our unit of length, and laying upon this equal squares till we have a perpendicular pile whose height is also equal to the unit of length. The *unit of volume is thus a cube whose sides are equal to the unit of length*: If we take 1 *cm.* as the unit of length, the unit of volume is a cube whose sides are 1 *cm.* long and will be called 1 **cubic centimetre** (*c.c.*). Similarly, we might have a *cubic yard*, a *cubic metre*, etc. To measure the volume of an object is simply to find out the number of these unit cubes needed to build it up.

§ 64. VOLUME OF A RECTANGULAR PRISM.—Suppose we have a rectangular prism whose length is 4 *cms.*, breadth 3 *cms.*, and thickness 2 *cms.* Evidently, we could begin to build up such a prism by placing 12 of our unit cubes in 4 rows of 3. This produces a block whose length is 4 *cms.*, and breadth 3 *cms.*, but which is only 1 *cm.* thick, and it is plain that such a block has a *volume* of 12 *c.cs.* By placing a second layer of unit cubes upon the top of this one, we should get a block of 4 *cms.* length, 3 *cms.* width, and 2 *cms.* thick as required. This contains 24 of our unit cubes. Its *volume* is 24 *c.cs.* Similarly, a rectangular prism of the same length and breadth, but 10 *cms.* thick, would have a volume of 120 *c.cs.*, and so on.

In a word, the *volume* of a rectangular prism, *expressed in c.c.*, is obtained by *multiplying the area of the base in sq. cms. by the thickness* (or height) *in cms.* If the prism be a cube its length, breadth, and height are equal, and the volume, for example, of a cube whose side is 5 *cms.* long is 125 *c.cs.*, since 5 rows of 5 unit cubes would give a prism the area of whose base is the same as that of the cube, and 5 such prisms superposed would reproduce the cube itself.

§ 65. MULTIPLES AND SUBDIVISIONS OF UNITS.—There will now be no difficulty in constructing a table to express the various multiples and subdivisions of the *cubic yard* and *cubic metre.* Thus—

1 CUBIC YARD = 27 *cubic feet* = 46656 *cubic inches.*
1 *cubic foot* = 1728 *cubic inches*, etc.
1 CUBIC METRE = 10^3 *cubic decimetres* = 10^6 *cubic centimetres* = 10^9 *cubic millimetres.*
1 *cubic decimetre* = 10^3 *cubic centimetres* = 10^6 *cubic millimetres.*
1 *cubic centimetre* = 10^3 *cubic millimetres*, etc.

The multiples, such as a *cubic mile* or a *cubic kilometre*, are similarly found.

§ 66. RELATION OF BRITISH AND METRIC UNITS.—The student should also calculate from the results of Exercise V. (§ 16) the relation between a *cubic yard* and a *cubic metre*, a *cubic foot* and a *cubic decimetre*, a *cubic inch* and a *cubic centimetre.* In doing this, the warning given in § 46 must be carefully borne in mind. The lengths found in Exercise V., depending as they did on estimation, will, when multiplied together, give areas which certainly cannot be trusted beyond two places of decimals (if so far) and when the multiplication is *repeated* to find the volume, the value of the decimal places becomes still more doubtful. In dealing with such numbers as are *doubtful in the second place of decimals* the student should, after the first multiplication, strike off decimal places up to the second from the area found (adding 1 to the second if necessary), and again give the final result, the volume, to *at most* two places, and better to one. When the lengths have been all measured so that the second place is *certainly* right, then three places may be retained in the volume.

The results of the calculation should be compared with those given below—

1 CUBIC YARD = 0.764 *cub. met.* 1 CUBIC METRE = 1.308 *cub. yds.*
1 *cubic foot* = 28.317 *cub. decims.* 1 *cubic decimetre* = 0.0424 *cub. ft.*
1 *cubic inch* = 16.387 *cub. cms.* (*c.cs.*). 1 *cubic centimetre* = 0.061 *cub. in.*

§ 67. THE LITRE.—It will be noticed that many common British volume measures, the *pint*, *quart*, *gallon*, etc., are not included here. The reason is that they do not bear any *simple* relation to the units of length. They will be considered later (see § 125). In countries where the Metric system is used, the common unit of volume is the *cubic decimetre* which has received the special name of *litre.*

§ 68. VOLUME OF ANY PRISM.—A little thought will show that the result obtained in § 64 can be extended so as to enable us to find the volume of any right prism. For if we find the area of the base of a prism, we know the number of *unit cubes* that must be packed together to cover that base, and if there be a fraction, we can imagine that a unit cube might be cut straight down without altering its height, so as to account for it. Then by taking a sufficient number of such layers, we could build up the prism to the required height.

Hence *the volume of any right prism* can be *found by multiplying the measure of the area of its base* (in any square unit) *by the measure of its height* (in the corresponding linear unit). The answer is expressed in the corresponding cubic unit.

If we have to deal with a *cylinder*, we can calculate the area of its base from the diameter (§ 56) and then multiply by the height.

§ 69. APPLICATIONS.—We can sum up what we have learned about the volumes of prisms in *general* terms as follows :—

(i.) *Volume of a right prism, whose base has an area,* a *sq. cms., and which has a height* h *cms.* $= a \times h$ *c.cs.*

(ii.) *Volume of a right cylinder, the radius of whose base is* r *cms. and whose height is* h *cms.* $= \pi r^2 \times h$ *c.cs.*

Exercise XXIV.

To measure the volumes of prisms of various forms and of a cylinder.

Apparatus.—Prisms, cylinder, glass scale, common callipers, slide callipers, micrometer.

Method.—(1) Find the area of the base (if rectangular, § 44; triangular, § 48; polygonal, § 50; circular, § 56). (2) Find the height. (3) Calculate the volume. (*N.B.* §§ 46 and 66.)

The following hints will be found useful :—(*a*) Let the measurements of lengths be as accurate as possible. Use the most delicate instrument that the size of the object will allow. (*b*) In dealing with a *triangular* prism there is no need to *draw* the perpendicular from apex to side of base. It can be measured either by setting the side against the fixed jaw of the slide callipers and bringing the movable jaw up to touch the opposite edge or, if the prism be too large for this, by laying the glass scale upon it so that one of the long graduations lies exactly along the side and the middle of the *mm.* graduations falls over the opposite corner. (*c*) If the base be a *regular polygon* with an *even number of sides* take advantage of the result in § 52. (*d*) If the base be an irregular polygon or, being regular, have an odd number of sides it may be copied very exactly upon paper or card and then measured.

§ 70. PYRAMIDS.—There is another set of bodies of regular shape, known as *pyramids* whose volume is easily measured. They differ from prisms in that, while in a prism both ends are alike, and the sides which join them are parallel, in a pyramid these sides are inclined to one another and meet in a point called the apex of the pyramid. This apex may be vertically above the middle point of the base, in which case we have a *right pyramid*, or the line joining the apex to the middle of the base may be inclined to the plane of the base, when we have an *oblique pyramid.*

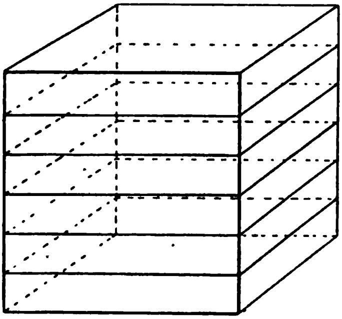

FIG. 27.

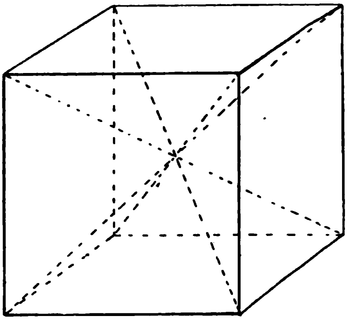

FIG. 28.

We shall deal only with *right* pyramids. A pyramid is described, from the shape of its base, as *e.g. a square pyramid, a right hexagonal pyramid*, and so on. We do not, however, speak of a circular pyramid. A right pyramid upon a circular base is called *a right cone.*

§ 71. VOLUME OF A PYRAMID.—We have now to discover how the volume of a pyramid is connected with its linear extension. This we can do as follows :—

Consider a cube each of whose sides is, suppose, 6 *cms.* long. Its volume will evidently be 216 *c.cs.*

Such a cube might be imagined as built up of six rectangular prisms each 6 *cms.* long, 6 *cms.* wide, and 1 *cm.* high (*Fig.* 27).

Each of these prisms will evidently have a volume of 36 *c.cs.*, *one-sixth* that of the cube.

But the cube might also be divided into six equal parts in another way. Imagine it divided as by the dotted lines in *Fig.* 28.

A cube divided in this way may be constructed as follows :—

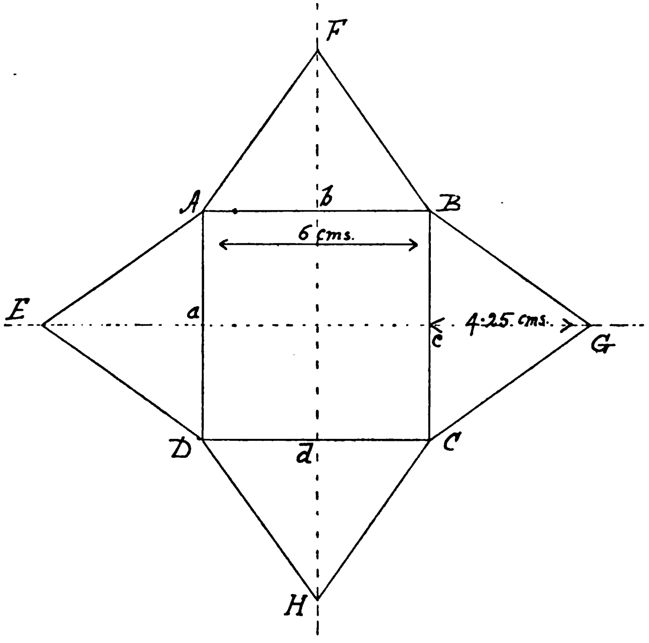

FIG. 29.

Exercise XXV.

To construct a pyramid that shall have one-sixth of the volume of a cube upon the same base.

Apparatus.—Cardboard, compasses, drawing-pen, set-square, ruler, steel *mm.* scale, sharp penknife.

Method.—Rule a square with sides exactly 6 *cms.* long, *ABCD*, *Fig.* 29. Mark the middle points of the sides *bcda*, and join *bd* and *ac*, producing both lines beyond the square as in *Fig.* 29. In the parts produced mark off points *F*, *G*, *H*, and *E* exactly 4.25 *cms.* from *b*, *c*, *d*, and *a*. Join each of these points to the nearest corners of the square.

Lay the cardboard upon a flat piece of wood and with a sharp-pointed knife cut out the figure *EAFBGCHDE*. Then lightly cut, so as not to go more than half-way through the cardboard, along *AB*, *BC*, *CD*, and *DA*. Now turn the cardboard over so that the letters are on the underside and bend up the corners so that *F*, *G*, *H*, and *E* meet in a point, and with strips of gummed paper fix them in this position. The resulting body will be a pyramid upon a square base.

It will be found that six such pyramids placed with their apices together and held in place by two india-rubber bands form a complete cube whose side is 6 *cms.* long.

Evidently each of the pyramids has a volume one-sixth that of the cube.

It is plain, then, that the pyramid just made has a volume of 36 *c.cs.* equal, that is, to the volume of a prism with a base of the same area and a height of 1 *cm.* Now the areas of the bases, and the volumes, being equal, it is evidently important to compare the heights. That of the prism we know to be 1 *cm.*, and we proceed to find that of the pyramid.

Exercise XXVI.

To measure the height of the right pyramid made in Exercise XXV.

Apparatus.—Glass scale, two rectangular blocks of wood with clean sharp edges, one of them nearly as long as the glass scale and, for choice, dark coloured.

Method.—Arrange the blocks as in *Fig.* 30 with the glass scale supported between them. Observe carefully, and record, the position of the upper edge of the lower block upon the scale. Now place the pyramid upon the lower block and move so that its image is clearly seen reflected in the glass scale. By lowering the eye till the apex of the pyramid appears exactly to coincide with its own image in the scale the height is easily read off.

Example of actual measurements—

Height of upper edge of lower block on scale = 3.23 *cms.*
Height of apex of pyramid block on scale = 6.25 *cms.*

∴ Height of pyramid = 3.02 *cms.*

Remembering that the tenths of *mms.* are estimated we may conclude that the pyramid is almost exactly 3 *cms.* high. Hence it is found that when a prism and a pyramid, upon bases of the same area, have the same volume *the pyramid is three times as high as the prism.*

It is true that we have only proved it for one particular case, but in a similar way it can be shown that this relation is always the same.

Hence *the volume of a right pyramid can be found by measuring the area of its base and its vertical height. It is the same as the volume of a right prism having the same area of base and only one-third the height.*

§ 72. APPLICATIONS.—To sum up what we have just found out—

The volume of a *right pyramid whose base has an area* a *sq. cms.* and whose *height is* h *cms.* $= a \times \frac{h}{3}$ or $\frac{1}{3}ah$ *c.cs.*

In the case of a *right cone* it is plain that this will be slightly altered.

If the *radius of the base be* r *cms.*, and the *height* h *cms.*, the *volume* $= \pi r^2 \times \frac{h}{3}$ or $\frac{1}{3}\pi r^2 h$ *c.cs.*

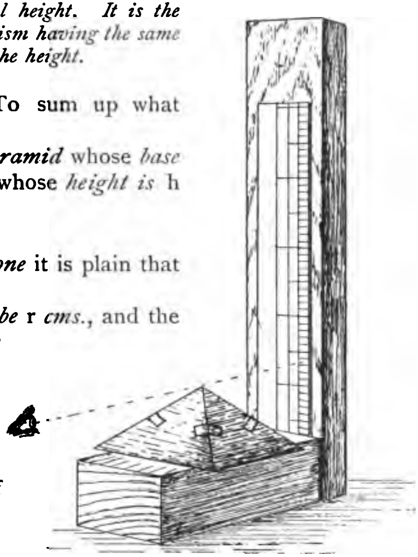

FIG. 30.

Exercise XXVII.

To measure the volumes of pyramids of various forms and of a cone.

Apparatus. — Pyramids, cone, glass scale, wood blocks used in Exercise XXVI., common callipers, slide callipers.

Method.—(1) Measure the area of the base as directed in Exercise XXIV. (§ 69) noting carefully the hints there given. (2) Measure the vertical height as directed in Exercise XXVI. (§ 71). (3) Calculate the volume. [*N.B.* §§ 46 and 66.]

Notice that the method of measuring a vertical height used here and in the last exercise is only another way of applying hint (*b*) of § 69. The scale cannot be laid close both to the edge and to the apex, for then it would not be vertical. So the scale is kept vertical and the error due to parallax avoided by observing when the point coincides with its image in a plane reflecting surface. The better to do this, glass scales are often made upon a strip of looking-glass. This plan, which is very often useful, depends upon properties of light which will be explained later in dealing with Optics.

§ 73. VOLUME OF CERTAIN OTHER OBJECTS.— It may be said, and with truth, that most of the objects we come across are neither prisms nor cylinders, pyramids nor cones, and hence that our discoveries so far do not help us much in finding the volumes of many of the commonest things. Indeed, we shall have to consider some other properties besides extension before we can actually find the whole extension, or volume, of bodies that have not a regular shape. But meanwhile it will be found that many objects are included among the classes named which hardly seem to belong to them at first sight. Thus *a straight piece of wire*, however thin, whose ends are properly cut or filed off "square," *is simply a right cylinder* which has a base of small area, and whose height is very great compared with its diameter. So also a circular disc of uniform thickness, like the microscope cover-glass used in Exercise XV. (§ 36), is really a right cylinder whose height is small compared with its diameter. Even a coin like a penny may be thought of as a cylinder, but since the ends are not plane we cannot find its volume simply by linear measurements. As another instance it may be pointed out that a rectangular piece of cardboard or paper, or a straight piece of steel "watch-spring" are simply *rectangular prisms* whose height is small compared with the length or breadth. The volumes then of such objects *can be found* by linear measurements, but it must be remembered that great care is necessary to measure the very small lengths involved as accurately as the larger ones. [Why?]

In many cases an object may be regarded as made up of two or more regular bodies. Thus a round lead-pencil sharpened to a perfect point (with a "sharpener") would appear as a right cylinder with a right cone upon one end of it. [What measurements would be needed to find its volume approximately at all events?]

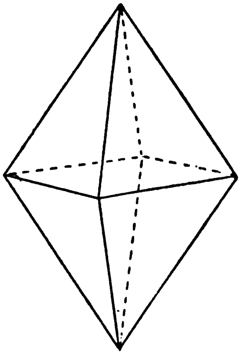

FIG. 31.

The most important cases of this kind are those in which the object considered can be regarded as made up of a number of pyramids. Very simple cases like those of the right *octahedron* (*Fig.* 31) need not be discussed at length. The object appears as two right pyramids exactly alike placed base to base, and the base is either a square or a rhombus. [What measurements would you make to find the volume and how would you calculate it ?]

But there are much more interesting cases than this. We saw, in § 71, that a cube may be regarded as made up of six equal pyramids whose apices meet in the middle of the cube. Now a cube has six equal faces which are *symmetrically* arranged, that is to say, are so placed that each of them has one other exactly opposite and parallel to it. Let us *suppose that we knew how to find the volume of a pyramid but did not know how to find that of a cube.* We could get at it, from what we knew of the pyramid, as follows. Each of the equal faces is the base of a right pyramid whose height is exactly half the distance between the middle points of opposite faces (see *Fig.* 28, § 71). We are supposed to know that the volume of a right pyramid $= \frac{1}{3}$(area of base × height), so the volume of each of the pyramids we are considering is $\frac{1}{3}\{$area of one face $\times \frac{1}{2}$(perpendicular distance between opposite faces)$\}$. Now for each face there is one such pyramid and therefore

$$\text{Volume of cube} = \text{number of faces} \times \tfrac{1}{3}\{\text{area of one face} \times \tfrac{1}{2}(\text{distance between opposite faces})\}.$$

Suppose the side of each face be l *cms.* long, then the area is $l \times l$ *sq. cms.* and the distance between two opposite faces is l *cms.* Hence, since there are six faces—

$$\text{Volume of cube} = 6 \times \tfrac{1}{3}(l \times l \times \tfrac{1}{2}l) = l^3 \text{ c.cs.}$$

Of course we knew this result before, and indeed our method of finding the volume of a pyramid in § 71 depended upon it. But this way of *working back* to it has taught us how to find the *volume of any object that is bounded by an even number of equal faces and can be pictured as made up of a number of right pyramids equal to the number of faces.* It will evidently be

$$\text{Number of faces} \times \tfrac{1}{3}\{\text{area of one face} \times \tfrac{1}{2}(\text{distance between opposite faces})\},$$

or, what is the same thing—

$$\tfrac{1}{6}\{\text{whole area of surface} \times \text{distance between opposite faces}\}.$$

We will apply this to one class of objects of perfectly regular shape, namely, *spheres.*

§ 74. VOLUME OF A SPHERE.—We saw, in § 53, that a circle must be regarded as a regular polygon with an enormous number of very short sides. Similarly, a sphere must be thought of as an object bounded by an enormous number of very small equal faces symmetrically placed round the centre. Each of these minute faces is imagined as forming the base of a right pyramid whose apex is at the centre of the sphere. Moreover, these faces are so small (being indeed points) that the height of each of these pyramids is simply the radius of the sphere. So, then, the distance between any two opposite faces is equal to the diameter of the sphere.

Thus the volume of a sphere will be

$$\tfrac{1}{6}(\text{area of its surface} \times \text{diameter}).$$

Now if the radius of the sphere be r *cms.*, its surface area is $4\pi r^2$ *sq. cms.* (§ 59) and its diameter is $2r$ *cms.*—

$$\therefore \text{ volume of a sphere, of radius } r \textit{ cms.} = \tfrac{1}{6}(4\pi r^2 \times 2r) \textit{ c.cs.}$$
$$= \tfrac{4}{3}\pi r^3 \textit{ c.cs.}$$

This result is very important because, where we have discovered another way of measuring the volume, we shall use it to prove what we have accepted on trust, that the area is $4\pi r^2$ *sq. cms.* (when the radius is r *cms.*). See also § 144.

§ 75. APPLICATION.—The measurement of the volume of a sphere thus involves only one linear measurement, that of the diameter. From this the radius and thence the volume can be deduced.

Exercise XXVIII.

To measure the volume of a sphere.

Apparatus.—Spheres, three rectangular wood blocks, glass scale, common callipers.

Method.—Measure diameter and calculate. The best way to measure the diameter will naturally vary with the size of the sphere. For moderate sizes the common callipers may be used (much as they were in Exercise XXII. § 56). For small ones the micrometer is better, while for large ones the arrangement shown in *Fig.* 32 is useful. The sphere is placed between the smaller blocks *a* and *b*, which are pushed close up to it and to the larger block *c*. By laying the scale across *a* and *b*, so that its graduations are exactly parallel to the edges, the distance between the inside edges, which is sensibly equal to the diameter of the sphere, can be read, by estimation, to 0.01 *cm*.

ADDENDUM

§ 76. AREAS OF SURFACE OF CYLINDER AND CONE.—These figures having been defined in §§ 62 and 70, we can now complete § 60 by considering what will be the whole areas of their surfaces. We have to think of the *right cylinder* as a right prism whose base is a regular polygon with an exceedingly large number of very short sides. [Why? see § 40.] Each of these imaginary sides corresponds to an imaginary rectangular face whose length is the height of the prism and its breadth the length of the supposed side. Hence the *area of the curved surface* of a cylinder is equal to the number of (imaginary) sides in its base × length of one of these sides × height, that is to say, is equal to *perimeter of base* × *height.* But the perimeter is here the circumference of a circle. Suppose the *radius of the base be* r *cms.* and the *height* h *cms.*, then the perimeter is $2\pi r$ *cms.* (§ 56) and the *area of the curved surface is* $2\pi r \times h$ *sq. cms.* But the *whole area* is made up of this, together with the areas of the two ends (each of which is πr^2 *sq. cms.*), and is therefore $(2 \times \pi r^2) + (2\pi r \times h) = 2\pi r(r + h)$ *c.cs.* Turning to the *right cone* we must think of it similarly as a right pyramid whose base is a regular polygon of very many short sides. Each of these imaginary sides corresponds to a tiny isosceles triangular face the area of which is $\frac{1}{2}$(length of (imaginary) side × "vertical height" of triangle). It is plain that since the imaginary sides are so small as to be mere points, the "vertical height" in question will be the same as the length of the equal sides of the isosceles triangles, that is, will be the *slant height* of the cone. Thus, we find, the area of each face

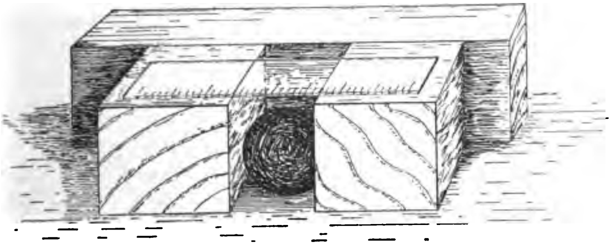

FIG. 32.

would be $\frac{1}{2}$(length of imaginary side × slant height). That of all the faces would therefore be $\frac{1}{2}$(circumference of base × slant height). Thus if *a right cone* have a *base of radius r cms.* and a *slant height of s cms.* the area of the slanting surface will be $\frac{1}{2}(2\pi r \times s) = \pi rs$ *sq. cms.*, and if to this we add the area of the base (πr^2 *sq. cms.*) we have for the *whole area* $\pi r^2 + \pi rs = \pi r(r+s)$ *sq. cms.*

§ 77. SYMBOLICAL NOTATION.—The student should now collect together in the form of a table all that has been discovered in working through Chapters IV. and V. about the relations between the linear extension of regular objects and their area and volume. In doing this he may, of course, shorten the statements by writing, say, *l cms.* for the length of an object, *but it must be clearly stated what every letter used stands for and in what kind and size of unit each measure is expressed.* These shortened forms are intended merely as convenient reminders of what has been learned by observation and experiment. To write or say such things as

"Volume of a pyramid $= \frac{1}{3}ah$"

is, of course, utter nonsense, for as they stand *a* and *h* might mean anything or nothing. What we found out in § 71 was that *a right pyramid whose base has an area of* a *sq. cms.* (or *sq. ins.* or any other square unit) *and whose height is* h *cms.* (or *ins.* or any other *corresponding* linear unit) *has a volume of* $\frac{1}{3}$ah *c.cs.* (or *cub. ins.* or any other *corresponding* cubic unit). Now the statement described as nonsense does not *necessarily* mean this.

The shortest way of stating the case clearly and accurately would be

"Volume of right pyramid $= \frac{1}{3}ah$ *c.cs.* when $a =$ area of base in *sq. cms.* and $h =$ height in *cms.*,"

it being understood, of course, that we might use other corresponding units if we chose.

CHAPTER VI

MASS

§ 78. POSITION.—We have now considered the property, chosen in § 2 and defined in § 3, called extension in space, and have seen that in measuring it we have to take into account two things: (i.) The distance in straight lines and in particular directions between certain points on the object, and (ii.) the shape of the object. It will be plain that in doing this we have made use of a notion which comes from our common experience, and which we express by the word *position.* In stating the shape of an object we are really describing the position of its extremities, points, edges, or faces, as the case may be (compare §§ 4, 59, and 73). Now any one who takes the trouble to think about it will recognise that when in everyday life we wish to describe the position of a point, we begin by saying that it is a certain distance *from some other point.* In fact, no one can even imagine the position of a point except with reference to some other point. And similarly in measuring, when we wish to find, say, the length of an object, we begin by *choosing some point* upon it, and then compare the distance between *this* and some other point upon it with the distances marked on our scales. So long as we are content to consider *only* the object we have chosen this is enough, but there is no reason why our curiosity should not lead us to inquire what is the distance between some point chosen upon our object and some other point upon some other object. Let us for a moment imagine that the distance between different points upon the object itself is of no importance. Then *so far as we are concerned*, for the time being that object is practically a point, and an object when considered in this way is spoken of as *a material particle.* If we chose two material particles we might measure the distance between them, and find the position of one relatively to the other. This, of course, is constantly done in everyday life. Thus "Bradshaw" tells us that the distance from London to Manchester (by rail) is 203 miles, and here London and Manchester are considered, for convenience sake, as material particles. To make the statement more exact we might add that the distance is measured from King's Cross Station in London to the Central Station in Manchester, but even so we should be regarding each of these large stations as a material particle. Notice that we do a precisely similar thing in our physical measurements. Thus in § 20 we copied a distance by means of compasses. Now the "points" of the compasses are not really points; they are objects of considerable size. But we treated them as material particles, and assumed that the distance between them was the distance between two points.

But distance is not all that is needed to define the position of one material particle with respect to another. Thus "Bradshaw" tells us that the distance from London to Harrogate is also 203 miles, but we know that the position of Harrogate is not the same as that of Manchester. In fact, we always add to our statement of distance a further statement of the *direction* in which that distance is measured. It is well known that in the cases chosen we should refer the directions to two fixed directions called "North and South" and "East and West," and speak of Manchester as being 203 miles (by rail) north-north-west from London, and of Harrogate as being 203 miles (by rail) north by west from London. Of course, these numbers and directions are only approximately correct, and we shall merely regard them as a rough illustration of the fact that *the position of a material particle* (that is, of any object we choose to consider for the moment as a point) *can be fully stated if we know* (i.) *the distance between it and some other material particle chosen as a point of reference, and* (ii.) *the direction in which that distance is measured.*

§ 79. REST AND MOTION.—Suppose now we have chosen any number of material particles, and are going to confine our attention entirely to them. These material particles form *a material system* which is the object of our investigation. For the sake of simplicity, let our material system contain two material particles. One of these particles we must choose as our point of reference. If the *position of the second particle does not change* (that is to say, if *neither* its distance from the first *nor* the direction of that distance changes) *during any period of time, then it is said to be "at rest" relatively to the first. If the position does change* (that is, if *either* the distance *or* its direction changes), *then the second particle is said to be "in motion" relatively to the first.*

The terms *at rest* or *in motion* are, then, purely *relative.* Thus if two bicyclists were riding along at *exactly* the same speed, then, considering each as a material particle, the position of one with respect to the other might not change in the course of, say, two minutes. We should say that either bicyclist had been *at rest with respect to the other* during those two minutes. If, however, we were to *extend our material system* so as to take in a neighbouring house, then making the house our point of reference, the position of either bicyclist with respect to it would change in the course of two minutes, and we should say that either was *in motion relatively to the house.* Now in common language we do not usually add the words which state relatively to what an object is at rest or in motion. Thus we speak of "a train in motion." This is because in that case our material system consists of the train and the earth, and by common agreement we regard the earth or some point of its surface as a fixed point of reference, so that by "a train in motion" is meant a train whose position relatively to some point of the earth's surface is changing. We will now consider what everyday experience teaches us about rest and motion.

§ 80. FIRST LAW OF MOTION.—In the first place, common observation tells us that if an object be at rest relatively to another it remains at rest except it be interfered with. If you place a book in a certain position relatively to a table you expect to find it in the same position later on, and if its position has changed you are confident that some one must have moved it. Of course experience is the *only* ground for believing this, but it is a very strong one and there are numberless examples of the confidence that we daily place in this particular property of objects. The milestones along a road were set down a mile apart, it may be as long as a hundred years ago, but observation having shown at the outset that each was at rest relatively to the surface of the road, people have gone on ever since believing that they remain at rest, and that the distance between each stone and the next is still a mile. From time to time it will have been shown in the course of surveys that this confidence is not misplaced.

Not only, however, do objects that are at rest remain at rest if they are let alone, but it seems at first sight as if even when an object is in motion it "strives," so to speak, to come to rest. If a cricket-ball be rolled across an ordinary meadow it goes some way and then stops, and would, so far as we know, always remain in the position where it comes to rest. From this and hundreds of similar simple examples we might be led to think that not only have objects the property of staying where they are, but that even when they change their position they do so unwillingly as it were, and after a while lose their motion and come to rest. But a little thought shows that this second conclusion is not really all that our common experience teaches us. For any one who has tried, say, rolling a cricket-ball, will agree that if, instead of rolling it across an ordinary field, we had rolled it across a good county ground, we should have found that without taking any more trouble we could cause the ball to roll very much farther, and should also notice that it continued to roll in almost exactly the direction in which it was started. Now imagine we could take the ball to a large sheet of smooth ice and roll it in a precisely similar way. We should expect that it would roll much farther still, and that its course would be still straighter, and the experience of people who have played hockey on the ice fully bears this out. But since we are supposed to have used the same ball in each case, and to have taken just the same amount of trouble in rolling it, we are obliged to conclude that its stopping depends, to a large extent at any rate, upon the surface over which it is rolled. The smoother the surface the farther the ball would roll, that much experience assures us, and so we may well doubt whether our supposed conclusion is sound, and whether the ball itself really "tries to come to rest" at all.

We may get some further idea of whether an object when moving "strives" to stop by considering how the motion of a cricket-ball appears to another person towards whom it has been rolled or hit. Every one who has fielded knows that it needs a great deal of effort to stop a cricket-ball, especially before it has gone far from the bat, that is, before it has been interfered with by rolling upon the surface of the ground.

We are led by this illustration to think that so far from stopping of its own accord an object that is moving goes on moving except it be interfered with by other objects.

And this surely is the teaching of all our experience. A man riding in a train, though at rest relatively to the train, is in motion relatively to the ground. And if the train be stopped suddenly, by the brakes or through a collision, the man does not stop. He goes on moving and is, as we say, hurled across the carriage. What is more, an object continues to move in a straight line. The occupants of a railway carriage when the train swerves suddenly round a curve to the left keep, for a moment, moving straight on and so, as we say, lurch to the right.

Thus we must conclude from our common experience that an object at rest remains at rest, an object in motion continues moving straight on, except in so far as it is interfered with.

This conclusion was first stated more than two centuries ago by Sir Isaac Newton. He called it the *First Law of Motion* and put it into these words :—

"That every body perseveres in its state of rest or of uniform motion in a right line except in so far as it is compelled by forces to change that state."

§ 81. INERTIA—FORCE.—It will be noticed that in Newton's statement a name is given to the causes which alter an object's state of rest or of motion. They are called "*forces.*" We shall have again and again to use the word force and must define clearly what we mean by it.

We have agreed that so far as our experience goes every object has two properties—(i.) extension in space and (ii.) the property of remaining at rest or continuing in motion. To this second property the name **inertia** is given. Now "inertia" is simply the Latin for idleness, and is intended to convey the idea that objects do not begin nor cease to move of their own accord.

These properties, extension and inertia, do not belong merely to any one object or set of objects. They are properties belonging to all stuff that we can learn about by means of our senses. That stuff which has extension and inertia is called *matter*, and the objects which we have considered, and shall consider, are simply portions of matter. *Matter*, then, *has inertia.* It does not begin nor cease to move of itself. But we know that objects do continually move and stop, and hence there is something besides matter, something which overcomes its inertia, and that something is called *force*. **Force**, then, is that which causes any object at rest to begin moving, or which, if an object be moving, alters either the speed or the direction of its motion.

We shall learn more about it by and by, but, for the present, it will be enough to remember that when we exert ourselves to push or pull an object so as to make it change its position we are bringing force to bear upon that object. Now we know that we may pull hard or gently, and that an object which a strong man can easily move could perhaps hardly be moved by a little child, and so we have to bear in mind also that forces may be great or small, and that a force which can move one object easily may not have nearly so much effect upon another object. In short, our everyday experience teaches us not only that there is something which we have called force, but also that it is capable of being measured.

§ 82. THE MEANING OF "LAW."—Before we go on to apply this new idea there is another thing which claims attention. Newton called his summary of the teachings of experience the First "*Law*" of Motion. When we use the word "law" in everyday life we think of some rule, made by Parliament, that must be obeyed under pain of certain penalties. Now the word "Law," as used in the expression First Law of Motion, obviously does not bear this meaning. We have to find out what it does mean.

Observation teaches us that in nature things happen regularly; and we may by observing find out that things seem to happen according to some particular rule, just as a foreigner who knew nothing about cricket might, after watching a few matches, conclude that it was a "rule" of the game that when the ball hit the wickets instead of the bat the batsman was "out." But it is very unlikely that a foreigner could discover all the rules of cricket by merely watching the game. He might, for instance, watch county matches for years and never learn that a batsman can be out for "handling the ball." In a like manner it is hardly possible for us to discover all the "rules" which operate in nature. Each "Law" that we seem to find out is simply based upon what we have seen. At any time somebody *might* see something which no one else had noticed and which would make us alter one of them or add a new one. It is thus with the First Law of Motion. It simply sums up what, *so far as* *anybody has hitherto been able to see*, is a rule. It is extremely unlikely that anybody will ever discover anything that will cause us to alter it. Still it is not impossible. The *reason why* matter has inertia we do not know. A body *does not* remain at rest or continue to move in a straight line *because of* the First Law of Motion. In common language a "*law*" says "*this shall be*" or "*this shall not be.*" In the language of Physics, and of all Science, a "*law*" says "*this will be*" or "*this will not be.*"

There is no mistake more common, and none so absurd, as to confuse the laws of nature with the causes of the things that happen in nature. The student should never forget that these laws are matters simply of experience, that they do not tell us *why* things happen, but only *how they seem to us to happen*, and he should not only be careful to avoid this ridiculous mistake himself, but should examine most carefully any statements about the laws of nature that he may come across to see whether it has been made.

§ 83. MASS.—Let us now return to our consideration of inertia and force. We have seen that our experience leads us to recognise that forces, understanding by that (for the present) the efforts that we can ourselves put forth, may be of different magnitudes and produce different effects when brought to bear on different objects. Thus suppose we have in a room a piano and a chair both standing on castors. We find that it needs a much greater effort to push or drag the piano about than is needed to do the like to the chair. In other words, we have to apply greater force to move the piano, to keep it moving, or to stop it in the place we wish. The piano, then, displays more inertia than the chair, and we naturally inquire what other differences there are between the two. The idea at once suggests itself that the piano has a much greater extension in space than the chair, and that possibly this may account for its showing more inertia. Further, experience tells us that while an object with large extension does very often show a great deal of inertia, this is not always the case. For instance, it sometimes happens that for want of a proper cricket-ball, made of leather and stuffed with cork, a "composition" ball, made mainly of cork-dust, having the same diameter (that is to say, the same extension) is used. Any one who has played with such a ball knows that it is easier to drive it and easier to stop it when driven. In other words, more force has to be applied to a proper cricket-ball to produce a certain effect than is needed for a "composition" ball, so that the latter, though having the same extension, displays less inertia. Evidently, then, inertia does not depend merely upon extension in space. It is natural to conclude that the amount of inertia displayed by an object will depend upon the quantity of inert stuff (that is to say, of matter) in it. This may be crammed into a small space or spread through a large one—if the quantity of matter is the same the inertia will be the same.

The *quantity of matter* in any object is called by a special name. It is known as the *mass* of that object.

Here, then, is another property which we may choose for measurement.

§ 84. UNIT OF MASS.—To measure mass, or quantity of matter, we shall need a unit. We have just seen that mass is a property that does not *necessarily* depend upon an object's extension, and so we shall be prepared to find that we are here called upon to choose a new *fundamental unit* (see § 43). As in our choice of another fundamental unit, that of length, we shall naturally choose one that is generally agreed upon, so that our measurements may be understood. There are two units of mass in common use.

The *British unit* of mass is the quantity of matter in a certain piece of platinum called the **Pound.**

The *Metric unit* is the quantity of matter in a certain other piece of platinum called the **Kilogramme.**

These two units are not of the same size, and we shall discuss them more fully in §§ 94 and 125.

The units having been chosen it becomes a question of copying them, for there can clearly be only one standard pound, and only one standard kilogramme. And here we meet with a difficulty. From what we have seen so far, it will be admitted that any other object displaying the same inertia as the particular piece of platinum called the standard pound would contain the same quantity of matter as this, that is to say, would have a mass of one pound. But we have as yet found no means of measuring inertia exactly. We have seen that our "muscular sense" enables us to tell when the inertia of one object is much greater than that of another, but that is all. And it is extremely difficult to measure inertia more accurately. Fortunately there is a simple way of avoiding this difficulty which we will now consider.

§ 85. GRAVITATION.—Every one knows that it is much easier to push or roll a large object along a smooth level surface than to lift it up bodily. A porter will easily drag along a station platform a box which he could not lift on to his shoulder. Now what is the difference between pushing or rolling an object along a level surface and lifting it up from that surface? In both cases the object is moved, and it is necessary to apply some force to overcome its inertia. But why should this force need to be so much greater when the object is moved up than when it is moved along? To understand this we must ask ourselves carefully what we mean by "up" and what by "along."

Every one is supposed to know that the earth on which we live is not flat, and there are abundant proofs that it is in shape nearly, but not quite spherical, being slightly flattened at what we call the North and South Poles, and somewhat more so at the North than at the South. This flattening is not very great, and for our present purpose we may think of the earth as being a large sphere about 12,700 kilometres (7900 miles) in diameter.

Now imagine two men, the one in London and the other on the opposite side of the earth, say at Melbourne, and suppose each of them to be engaged in lifting some object. Each finds it easy to roll his object *along* but hard to lift it *up*. Now think what each means by "up." For the man in London from *a* to *b* (*Fig.* 33) is "up." For the man in Melbourne from *c* to *d* is "up." But the direction *a* to *b* is opposite to the direction *c* to *d*, and hence we see that the term *up* is a *relative* one.

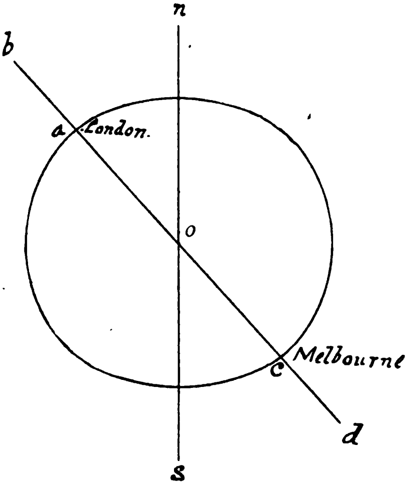

FIG. 33.

Both men in lifting the object *up* would move it *farther from* the point *o*, *the centre of the earth*, while in rolling it along its distance from *o* is not altered.

When, therefore, we speak of moving an object *up* or *down*, we mean so moving it that its distance from the centre of the earth is increased or diminished, while by moving it *along* we mean so moving it that its distance from the centre of the earth is not altered.

What the experience of everyday life teaches us is that it is much harder to move an object farther away from the centre of the earth than to move it so that its distance from the centre remains unaltered. And it teaches us more than this. It is not unusual to see, at a railway station, one or two porters pushing along a large carriage or a loaded truck which it would be quite impossible for them to lift up at all. So that a force which will easily overcome the inertia of the carriage is altogether insufficient to lift it up. In short, we conclude from this and thousands of similar cases that in moving a body "up" there is more to be done than merely to overcome its inertia. And we know this from another fact too. When a railway carriage is pushed along as above, the porters have first to apply a force to overcome its inertia and set it in motion, and secondly, remove this force and apply one in the opposite direction to bring it to rest again. Once at rest it will remain so till another force is applied to it. But it is different with an object lifted up. Not only must a force be applied to overcome its inertia, and that a much greater one than would be needed to move it along, but if it is to remain at rest above its original position it must be, as we say, held up. If the original force be removed altogether the object will not remain at rest, it will fall down again. In other words, with respect to *up and down* motion, all objects behave *as if they were already acted on by a force* which urges them nearer to the centre of the earth. And finally, our experience teaches us that the greater the inertia of an object, as judged by the effort it costs us to roll or push it along a smooth level surface, the greater is this force which pulls it towards the earth's centre and resists our attempt to lift it up.

It was again Sir Isaac Newton who first recognised these conclusions. He took into account not only the behaviour of objects such as we have considered but also those of the moon and of the earth itself and the other planets, and, just as we have to conclude from our experience that the big portion of matter we call the earth attracts or draws towards itself other portions of matter, so Newton was able to conclude from more extensive observations that "*every portion of matter attracts every other portion of matter.*" . . . But Newton went further than this. He concluded that the force with which one portion of matter attracts another depends upon (i.) the quantity of matter in each, that is, the *mass* of each, and (ii.) the *distance* between them. Now Newton had the observations of astronomers to help him in coming to this conclusion, and by applying mathematical reasoning to them was able to discover far more than our common experience teaches us. We can, however, say this much, that the greater the mass of an object the greater is the force with which the earth attracts it. We do not know from everyday experience that *every* portion of matter attracts *every other*. However, an experiment was made about a hundred years after Newton's time by the Hon. Henry Cavendish which, though it is too delicate for us to perform, is very easily understood, and which not only proves that one small portion of matter attracts another, but also makes it possible to see how the magnitude of this force of attraction depends on the masses of the objects and on the distance between them.

The attraction of one portion of matter for another is called *gravitation* (see also § 90).

§ 86. CAVENDISH'S EXPERIMENT.—No attempt will be made here to describe the experiment exactly. We shall merely try to gain an idea of what was done. Imagine a thin rod having at its ends two small equal spheres of lead or other substance, and suspended by a wire from a fixed support so as to hang perfectly horizontally. If this rod turns to one side it twists the wire, and this, untwisting itself again, drives the rod back. The rod once set vibrating will thus swing to and fro, and by causing it to oscillate inside a divided circle it is possible to observe, with suitable precautions, the extreme points of its swing to either side. The middle of this swing is where the rod would rest when still, and is called the position of equilibrium (*Fig.* 34, *a*).

Now suppose we have two large balls (Cavendish used leaden spheres a foot in diameter) and place them so that their centres are in the same plane as those of the small spheres and at a known distance from the position of equilibrium, as in *Fig.* 34, *b*, and that the rod is again set swinging.

If these larger spheres attract the smaller ones the swing will now be a little longer in the direction indicated by the arrows and a little shorter in the opposite direction, and the position of equilibrium will be shifted in the direction of the arrows. Similarly, if the large spheres be placed as in *Fig.* 34, *c*, the swing will be a little longer the other way, and the position of equilibrium will be shifted in the opposite direction.

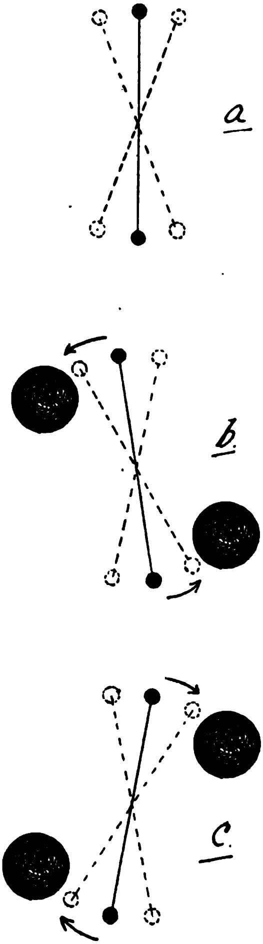

FIG. 34.

This is what Cavendish found to be the case.

From the amount of the shifting, that is to say, of the twist in the wire, it is possible to calculate the force of attraction between the large and small spheres.

This apparatus, called the *Torsion Balance* (from Latin *torquere*, to twist), has been greatly improved since the time of Cavendish, especially by Professor C. V. Boys, who has shown how to use a very fine suspending fibre made of quartz which untwists itself much more readily than a wire, and by which much smaller forces can be measured. An account of his work will be found in *Nature*, vol. l. p. 330, etc. (see also Poynting, *Nature*, vol. lxii. p. 403).

The outcome of experiments with the Torsion Balance, in which spheres of various sizes and of different materials have been used, is to show that any portion of matter attracts any other portion of matter, and that the force of this attraction varies directly as the product of the masses of the two portions of matter, and inversely as the square of the distance between their centres.

Thus, if we were to agree that the force with which two masses, each of 1 pound, placed with their centres 1 foot apart, attract each other should be called 1 unit of force, then the attraction between two masses of, say, 5 pounds and 8 pounds, whose centres were 4 feet apart, would be $\frac{5 \times 8}{4^2} = \frac{40}{16} = 2.5$ units of force. We do not actually take the force with which two unit masses, having their centres unit distance apart, attract one another as the unit of force, because it is exceedingly minute. Indeed, the force with which two masses, each of 100 tons (22,400 pounds), placed with their centres 1 foot apart (if that were possible), would attract one another would hardly be more than enough to lift up 2 pound masses.

§ 87. LAW OF UNIVERSAL GRAVITATION.—This "Law" (see § 82) which Newton stated about 1687, and which has thus been confirmed by later experiments, was as follows:—

> *Every portion of matter attracts every other portion of matter with a force whose direction is that of the straight line joining the two, and whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.*

As we have seen, Newton had not the results of experiments with the Torsion Balance to help him in recognising this rule. He arrived at it by a method which is often followed in science, and which it is of the utmost importance to understand.

§ 88. HYPOTHESIS.—The facts which Newton had to go upon were: (i.) the circumstance, known to everybody, that all objects upon the earth appear to be attracted with considerable force towards the earth's centre; (ii.) the observations of astronomers concerning the distances and movements of various heavenly bodies.

These observations were principally those of the distances of certain planets from the sun and of the times they take to go round it, which had been made by a Swedish astronomer, Tycho Brahe, about 1580. Tycho Brahe had a disciple, a German named Johann Kepler, who, after his master's death in 1601, continued carefully to examine the numbers they had found, and in 1619 stated three "Laws" or rules, of which the most important is known as "Kepler's *Third Law*." It was to the effect that the square of the time taken by a planet to go round the sun always bears the same relation to the cube of its average distance from the sun.

Newton took these laws of Kepler's, and, arguing from them, was able to arrive at this conclusion :—

> *If* a planet were drawn towards the sun by a force similar to that which draws objects towards the earth, and
>
> *If* that force depended directly upon the product of the masses of the sun and the planet and inversely upon the square of the distance between them,
>
> *Then* the square of the time in which that planet goes round the sun *would bear nearly* a constant proportion to the cube of its average distance from the sun.

But Kepler had stated that it bore a proportion which was *quite* constant. Now Newton's calculations showed him that *if his suppositions were correct*, the proportion would be a little less with the large planets like Jupiter and Saturn than with the small planets like the earth. More accurate measurements showed that this was indeed the case, and so Newton's suppositions were likely to be true.

A supposition of such a kind, made with a view of seeking for some connection between the various properties of objects, is known as a **hypothesis** (a Greek word meaning something laid down or assumed).

What we have called the Law of Universal Gravitation was, then, in the first place a *hypothesis*. It was assumed as something which *might* be true, and the *question* was asked: What would follow? We have seen that the *answer* was: A certain relation will be found to exist between the time a planet takes to go round the sun and its average distance from the sun. But it was possible to *test* this by making measurements, and it was found that the relation in question does exist.

A large number of similar tests were applied by Newton. If the hypothesis were good, then many details could be foretold about the movements of the moon round the earth. By observing the moon's movements these predictions were all shown to be true, and so, gradually, Newton and other men of science came to be more and more convinced that the original hypothesis which enabled these things to be foreseen must itself be true.

From the time of Newton until now, in spite of the enormous number of new heavenly bodies that has been discovered, no case has occurred in which the motion of any one of them has been other than that which can be foreseen by assuming the " Law of

Gravitation." It is true that sometimes difficulties have arisen, but they have only led, in the long run, to a firmer belief in Newton's hypothesis. For example: In 1781 Sir William Herschel discovered that the heavenly body which we know by the name of Uranus, and which had up to then been supposed to be a fixed star, is in reality a planet revolving round the sun at an immense distance. The motion of this planet was carefully watched and its path or orbit was mapped out, so that astronomers expected to be able to predict where Uranus would be found at any particular time. For some fourteen years the planet followed this path, but about 1795 it was noticed that Uranus was in front of the place where it was expected to be, and for the next twenty-seven years it continued to gain in this way. After that it began to lag behind again. Now alterations of this kind are not foreseen by Newton's hypothesis. One of two things must follow. Either Newton's hypothesis is incorrect, or there is some other force, whose existence had not been taken into account, acting upon the planet Uranus. Two young astronomers, M. Urbain Leverrier, of Paris, and Mr. J. C. Adams, of Cambridge, independently tried to find some possible origin for such a force, and both saw that it might be due to the attraction of another planet, not yet discovered, still farther away than Uranus, and which in the course of its journey round the sun was sometimes in front of Uranus, pulling it on, and sometimes behind, retarding it. By most difficult calculations they both concluded, towards the end of 1845, that if such a planet existed it ought to have a certain orbit which they described. Within a year Dr. Galle, of Berlin, discovered this planet, now called Neptune, close to the predicted place. Now the calculations of Adams and of Leverrier had assumed that the hypothesis of universal gravitation as expressed in Newton's "Law" was correct, and hence the peculiar movements of Uranus, so far from overthrowing the hypothesis, have been the means of increasing the confidence with which it is regarded.

§ 89. THEORY.—The piece of history just sketched is very valuable, because it gives an idea of the nature and use of hypotheses in science. A hypothesis begins by being a mere supposition, suggested by some observation or other. It is put to the test by inquiring what must follow if it be true, and then making experiments to see whether the predictions are in accordance with facts. If they are the hypothesis is good, and the more numerous the cases of agreement the better the hypothesis.

We may have several simple hypotheses, each of which has its own set of conclusions duly tested by experiments, and which are grouped together to form a larger hypothesis, leading to wider conclusions that are tested by further experiments. A group of hypotheses of this kind forms what is called a **theory.**

It will be seen that we ought, strictly, to have spoken not of *Newton's* "hypothesis" but of his "theory of universal gravitation," for there are at least four suppositions which are stated in pairs for the sake of brevity in § 88. [What are they?]

A theory is a complex hypothesis, made up of simpler ones, each of which must be good for the theory to be of any use.

Finally, we learn from the story of the discovery of Neptune, that an apparent failure of a theory coming after a great number of successes *may* mean that there is some new fact waiting for us to discover it. It will be seen, then, that theories have been, and are, of the greatest value as aids to the finding out of facts.

There are two mistakes against which the student of science must always be on his guard.

The first is that of imagining the making of theories to be the end and aim of science. The aim of science is to observe accurately and to discover, if possible, the connection between the different facts observed. The use of theories is only one of the means, and when properly carried out one of the most valuable means of pursuing this aim.

The second mistake is that of confusing theories with facts. Facts we learn through our senses, and though these senses are easily deceived, yet, as we have seen in Chapter II., it is possible by taking precautions and training our faculties to avoid errors and get fairly accurate results. But theories are not the direct results of observation; they are founded upon a *supposed* connection between facts, and it is always *possible* that the discovery of some new fact may compel us to abandon even the best of theories. There was a theory once, propounded by Sir Isaac Newton, which supposed that what we call light consists of exceedingly tiny particles of matter shooting out with enormous speed from luminous bodies. It was not a specially good theory, but for more than a hundred and twenty years no fact was observed which actually contradicted it. Then it was found that light travels more slowly through water than through air. Now if the supposition of the "emission theory" had been correct, light must have been found to travel more rapidly through water than through air. So the theory of emission was given up, and since 1815 another theory altogether has taken its place, which *up to the present* has not come into collision with observed facts.

§ 90. WEIGHT.—To return now to the subject of gravitation. So far as objects like the moon and planets which are beyond our reach are concerned, we have to be content with a theory, a theory, however, which is the best that science has hitherto produced, inasmuch as it has been more often verified through the success of its predictions than any other.

But when we are dealing with matter that we can handle, the attraction between different portions of matter is no longer inferred by complicated calculations, it is a question of simple experiment, and we have seen in § 86 that this attraction can be measured and shown to depend in a particular way upon the quantity of matter in each of the objects and upon the distance between them.

When one of the objects considered is the Earth, the force of attraction between it and the other object is called by a special name. It is known as the **weight** *of that other object.*

The student should notice carefully that this is a misleading way of naming it. It looks as if its weight were just as much one of the properties of an object as its extension in space or the quantity of matter in it. But the "weight of an object" is something that belongs jointly to that object and to the Earth. It is the force with which each pulls the other towards it. The Earth attracts the object, the object attracts the Earth, but since the mass of the Earth is very large and that of the object, by comparison, small, we *only notice* the effect of this force upon the object. That is why we speak of the "weight of the object."

There will now be no difficulty in seeing that this force called "the weight" will be proportional to the product of the mass of the object and the mass of the Earth, and inversely proportional to the square of the distance between the object and the centre of the earth. To put it shortly, if the mass of the object be m units, the mass of the Earth M units, and the distance between the centres of the object and the Earth be D units of length, then the force that is called the weight of that object will be *proportional to* $\frac{m \times M}{D^2}$ units of force.

We saw, in § 86, that the unit of force in actual use is much greater than the force with which unit mass attracts unit mass at unit distance. Let us say that it is K times as great.

Then the weight of an object whose mass is *m* units $= \frac{m \times M}{D^2} \div K$ units of force.

If we had another object whose mass was *x* units its weight would be $\frac{x \times M}{D^2} \div K$ (the letters having the same meaning as before).

Now strictly speaking, the object we are considering is part of the Earth, and so our letter *M*, instead of standing for the number of units in the mass of the whole Earth, stands for the number of units in the mass of the Earth *less* the number of units in the mass of our object. But the quantity of matter in the Earth is so enormous that *M is practically always the same.* As to our letter *D* which stands for the number of units of length in the distance between the centre of the earth and our object, it is plainly not always the same. In the first place, the shape of the earth is not regular (§ 85), and it has been found that the diameter between the poles is about 27 miles shorter than the diameter at the equator. In the second place, the surface is not even. Thus in moving from one part of the world to another we alter, noticeably, our distance from the centre. Evidently the nearer we get to the centre the greater will be the force with which any object is attracted, and hence the position of an object on the earth's surface affects its weight. It is found by experiment that a pound (§ 84) weighs least at the equator, and that its weight increases as it is brought nearer to the poles. But so long as an object remains at the same place, there are only local inequalities of the earth's surface to be considered, and these are very trivial compared with the immense distance to the centre. Hence *at the same place* the distance represented by *D is practically always the same.* It is plain that *K is always the same*, for it depends only on the unit of force chosen, and so the proportion between the weights at the same place of two objects whose masses are *m* and *x* units respectively is

$$\frac{\frac{m \times M}{D^2} \div K}{\frac{x \times M}{D^2} \div K} = \frac{m}{x}.$$

In words, *the weights of two objects at the same place are in the* *same proportion as their masses.* If the weights at the same place are equal, then the masses are equal also.

We shall now proceed to show that there are two means by which it is easy to tell when two forces are equal, and hence how, by comparing the weights of objects at the same place, we can readily compare their masses.

> [*Note.*—Seeing that one of the most striking results of the attraction between different portions of matter is the existence of this force which we have called weight, and remembering that the Latin for weight is "*gravitas*," the student will now understand why the attraction between any two quantities of matter is called *gravitation* (§ 85).]

§ 91. STRAIN AND STRESS.—When force is applied to a *fixed* object which it cannot therefore set in motion as a whole, experience shows that it may have another effect, namely, to alter either the shape or the volume of that object, or both. *Any alteration in the shape or volume of an object is called a* **strain.** When we are dealing with solid objects the alteration in volume produced by any force we can apply is usually very small indeed, and we shall not here concern ourselves with it. On the other hand, we know that it is frequently not hard to alter the shape of a solid body. An *alteration of shape only*, not accompanied by alteration of volume, *is called a shearing strain* or, simply, a **shear.** Now for some reason that we do not know every object withstands the attempt to alter its volume or its shape. In fact, an opposing force is developed in the interior of the object, and this opposing force is called a **stress.** If the strain to be withstood is a mere change of shape, a *shearing strain*, then the force which withstands it is called a **shearing stress.**

The property of withstanding change of volume or shape, or, in other words, of opposing a force called a stress to any strain produced by an outside force, is called *elasticity.*

All objects possess *elasticity of volume*, that is to say, every object opposes a stress to any force by which it is attempted to alter its volume. But *not all* objects possess elasticity of shape, or, as it is called, *rigidity.* With respect to shearing strain, or alteration of shape, there are two classes of objects :—

(i.) Those which can oppose a shearing stress to any outside force that produces a shearing strain. These *have rigidity* and are called *solids.*

Thus it is a perfectly familiar idea that the turning-power of a force about a given point depends not only upon the size of the force, but also upon the distance between the point where it is applied to an object and the point where that object is fixed. The greater this distance the greater the turning-power of the force about the fixed point. The turning-power of a force about a point is called, in physics, the *moment* of the force about that point.

Simple experiments with an ordinary gate teach us that *the moment of a force about a point depends upon* (i.) *the magnitude of the force;* (ii.) *the distance* between the place where the force is applied and that point. Suppose now we have an object fixed not at one extremity but in the middle, such, for example, as a turnstile. By pushing at one end the object can be made to rotate as before. If it be pushed in exactly the same direction but at the other end it will rotate in the opposite direction. If two people try to go through opposite sides of a turnstile in the same direction they try to turn it in opposite directions, and it may easily happen that though both apply a considerable force, neither is able to overcome the other and the turnstile remains still. This shows us that two forces whose directions are the same may, if applied to an object on opposite sides of a fixed point, have equal but opposite moments about that point, as proved by the object's remaining in equilibrium. If in such a case we knew that the two forces were exactly equal as well as in the same direction we could infer (since their moments depend as shown above upon (i.) their magnitudes and (ii.) their distances from the fixed point) that their distances from the fixed point must also be equal.

What is more important to us, if we knew that two forces acted in exactly the same direction and at equal distances on opposite sides of the fixed point and found that their moments were equal (that is to say, that the object did not turn), then we could be sure that the two forces were equal.

[It is obvious why we have insisted that the forces should be in the same direction, for the simplest experiment will convince one that an oblique push has less effect in opening a gate than a straight one, and if the directions of the forces were different this would affect the values of their moments.]

This gives us a second, and still more important means of comparing forces. We have only to cause them to act in the same direction upon the extremities of a rigid body supported exactly in the middle. Then if no rotation of the body occurs the forces are equal. If there be rotation its direction will show us which is the greater force.

It is plain how easily this may be applied to the comparison of weights. If we support a rigid bar at its middle point and hang the objects whose weights are to be compared on to the ends, we can tell at once whether these weights are equal or which of them is the greater. This is the principle of the *Common Balance*, which will be described more fully in § 100.

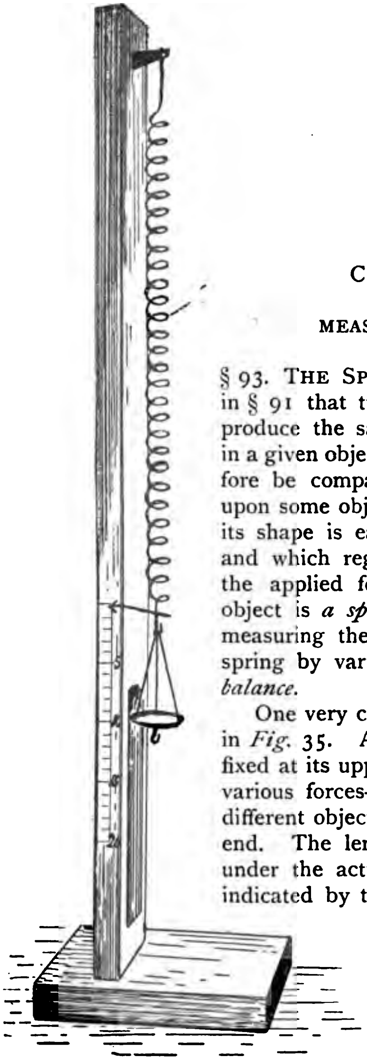

FIG. 35.

CHAPTER VII

MEASUREMENT OF MASS

§ 93. THE SPRING BALANCE.—It was seen in § 91 that two forces are equal when they produce the same amount of shearing strain in a given object, and that forces might therefore be compared by causing them to act upon some object such that any alteration of its shape is easily observed and measured, and which regains its original shape when the applied force is removed. Such an object is *a spring*, and an arrangement for measuring the shears produced in a given spring by various forces is called *a spring balance*.

One very common form of this is shown in *Fig.* 35. A coil of steel or other wire is fixed at its upper end to a rigid support, and various forces—for example, the weights of different objects—can be applied to the lower end. The lengthening of the spiral spring, under the action of these various forces, is indicated by the position of a small pointer that is attached to the lower end and moves over a fixed vertical scale.

A more convenient and accurate form is that shown in *Fig.* 36. It consists of a thin straight strip of steel, *aa′*

(a piece of clock-spring) which is fixed upon a wooden block *b* by means of a smaller block *c*, kept in position by the screw *s*. The spring projects horizontally, and at the end *a′* carries a scale pan, *p*, and a long light pointer *a′k*. Behind this pointer is placed a board *m*, upon which is fixed, by means of drawing-pins, a piece of "curve paper" ruled with lines 1 *mm.* apart, like that used in Exercise XXIII. § 57.

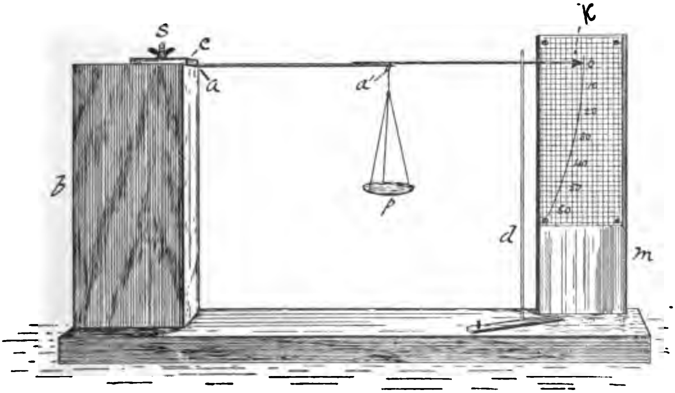

FIG. 36.

The *scale pan* is suspended, as shown in *Fig.* 37. Small notches, *n*, *n′*, are made with a file in the sides of the spring, about 2 *cms.* and 8 *cms.* respectively from the end, and at the end a deep groove *g*. A piece of wire, bent as in the figure, is laid upon the spring, so as to form a bridge across the end of the groove, and is firmly bound in position. A small double hook of steel

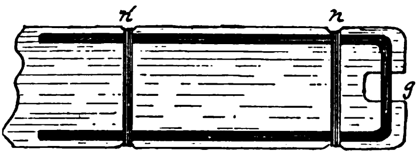

FIG. 37.

wire serves to suspend the scale pan—which may be a piece of cardboard suspended by four silken cords—from this.

It will be found convenient to have a *small* hook on the bottom of the scale pan, exactly in the middle (for reason, see § 140).

The pointer is suitably made of a reed, which carries a small arrow-head of blackened paper cut to a sharp point. This reed is easily fixed to the under side of the spring by means of some "Faraday's cement," and should be attached a little on one side of the middle, so as not to interfere with the hook from which the scale pan depends.

It is evident that if an object be placed in the scale pan its weight will produce a shear in the spring and cause the pointer to move downwards over the squared paper. By reading off the number of *mm.* divisions crossed a measure of this shear, *and therefore a measure of the force which causes it*, can be obtained.

It will be noticed, however, that the effect of placing an object in the pan is not actually to bring the pointer down at once into a definite position, but to set it vibrating. [Why? compare §§ 91 and 80.] After a long enough time it will come to rest, but it is better to stop the vibrations rapidly, and this can be done by "*damping*," that is to say, by touching the spring or the pointer *very lightly* with the edge of a piece of paper or with another reed. For convenience sake, a second reed may be mounted upon a piece of wood, secured by a small peg, so that it can be turned in and made *just to touch* the pointer for a moment and then turned out again before the reading is taken (*d* in *Fig.* 36).

The length of the projecting part of the spring and that of the pointer will be found to affect the displacement produced by a given weight in the pan, and must be so adjusted as that with the largest weight to be used the pointer shall not be depressed more than about 15 or 20 *cms.* across the squared paper.

It will be necessary to adjust the position of the spring once for all and then to *graduate* the balance, that is to say, to observe and record the deflections produced by *the weights of known masses* placed in the pan. For this purpose copies of the standard of mass, or fractions of it, are needed.

§ 94. STANDARDS OF MASS.—As in the case of the unit of length so in that of the unit of mass, different magnitudes have been adopted in different countries. In Great Britain the legal unit is now the mass of a certain piece of platinum known as the pound avoirdupois, or simply as the **pound.** This unit was in use for the weighing of merchandise in very early times, but it was only made the legal standard in 1834. Before that a smaller unit, known as the *pound Troy*, was the standard. This smaller unit is now only used in weighing gold and silver and some jewels.

The multiples and subdivisions of the pound in common use are as follows :—

437.5 *grains* = 1 *ounce*
16 *ounces* = 1 POUND (lb.)
112 *pounds* = 1 *hundredweight* (cwt.)
20 *hundredweights* = 1 *ton*.

It will be noticed that 1 *pound* (avoirdupois) *contains* 7000 *grains*. The pound Troy only contains 5760 grains.

In France, and in most other European countries, the Metric system is used. In this system the unit is the mass of a certain piece of platinum called the **kilogramme.** [See further, § 125.] This is subdivided decimally as follows :—

1 KILOGRAMME = 10 *hectogrammes*
= 100 *decagrammes*
= 1000 GRAMMES,

and

1 GRAMME = 10 *decigrammes*
= 100 *centigrammes*
= 1000 *milligrammes*.

It will be noticed that the names are arranged as if the *grámme* were the unit, and that the Greek prefixes kilo-, hecto-, deca- are used for the multiples, the Latin deci-, centi-, milli- being reserved for the subdivisions, as in the case of the metre (§ 8).

The only multiple of importance is the *quintal* = 100 kilogrammes.

The Metric system is now used in all physical measurements, and since the kilogramme has a large mass, more than twice that of the pound, it is convenient to regard the *gramme* as the practical unit.

Copies of the standard kilogramme or its subdivisions are contained in sets of so-called "weights."

These are usually supplied in a series of sizes beginning, for example, with 100 grammes and running down to 0.1 gramme or, in better sets, to 0.01 gramme.

As a rule, they are arranged as follows (the numbers stand for grammes) :—

100 50 20 20 10
5 2 2 1
0.5 0.2 0.2 0.1
0.05 0.02 0.02 0.01

The larger "weights" or standard masses (see *Fig.* 38) are made of brass and are in the form of short cylinders with small handles at the top. They are lifted by means of a pair of pincers and must not on any account be touched with the fingers. The smaller ones, from 0.5 gramme downwards, are either small pieces of platinum (or aluminium) foil stamped with the appropriate number, and having one corner turned up so as easily to be handled with the pincers, or else pieces of thick aluminium wire bent once or more at right angles, so as to be easily picked up from any position. These wire "weights" have no numbers stamped upon them (though their values are sometimes indicated by the number of arms), and their values must be learned by practice.

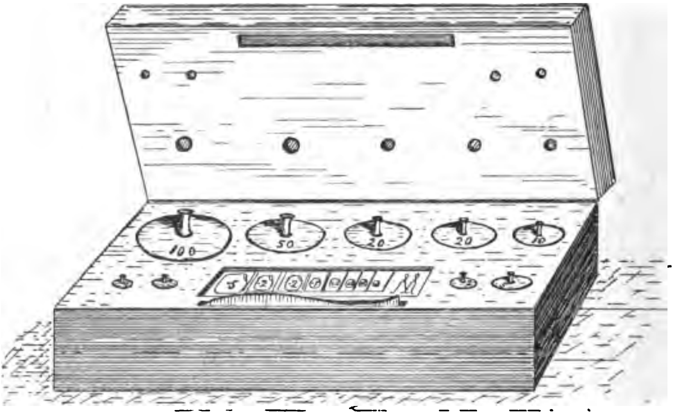

FIG. 38.

The student should study thoroughly and carefully the set of weights which he will have to use, noticing the value *and position* of each, so that he can without hesitation recognise which have been removed and **read the values directly from the empty spaces in the box.**

Once this has been perfectly mastered he may proceed to graduate the spring balance.

§ 95. GRADUATION OF A SPRING BALANCE.

Exercise XXIX.

To graduate a spring balance.

Apparatus.—Spring balance (as in *Fig.* 36), box of weights (including one of 100 grammes), curve paper.

Method.—The spring is fixed in position by means of the screw in the movable block, *s* (*Fig.* 36), and the position of the end of the pointer upon the squared paper is carefully noted and marked with a sharply pointed pencil. Then the 100 gramme mass is placed in the pan, the vibrations of the spring are damped as already directed, and the new position of the pointer upon the squared paper is noted as before. In making these readings the greatest care must be taken to avoid parallax. [What precaution is plainly necessary? See § 13.] The position of the pointer when the pan is loaded with 100 grammes should not be less than 15 nor more than 20 *cms.* below the zero line. If necessary, the projecting part of the spring should be slightly lengthened or shortened till the deflection falls within these limits. Once the spring has been adjusted so as to give a convenient deflection it should be kept permanently in position.

Then remove the 100 gramme "weight" and see that the zero position is still the same as at first and put the mark O against it. Now place the 10 gramme "weight" in the pan, mark the position of the pointer (after damping as usual) and number it "10 grammes." Proceed in this way to find the positions for 20, 30, 40, etc. up to 100 grammes. By joining the points so marked a curved line like that of *Fig.* 39 will be obtained.

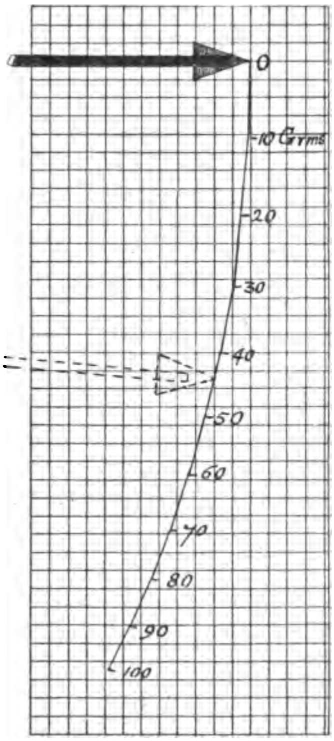

FIG. 39.

It will be seen that the number of *mm.* divisions crossed by the pointer diminishes with each addition of 10 grammes to the load. Thus in the experiment by which the curve of *Fig.* 39 was drawn the deflection 0-10 was 20.8 *mm.*, the deflection 40-50 17.2 *mm.*, and the deflection 90-100 only 12.9 *mm.* Hence it is not possible directly to read off the mass by counting the number of *mm.* divisions crossed, because the weight required to produce a deflection of 1 *mm.* continually alters as we proceed along the curved line.

We have now to find some means of determining at a glance what is the mass whose weight has produced a deflection of so many *mms.*

§ 96. REPRESENTATION OF VARYING QUANTITY BY A CURVE. —Our experiment with the spring-balance has given us two series of numbers. Firstly, a series of masses, 10, 20, etc.... 100 grammes; and secondly, a series of readings—for example, 20.8 *mm.*, 41.3 *mm.*, etc. . . . 169.4 *mm.*—which give the deflections produced by the weights of those masses. It is plain that the numbers of the second series (the readings) depend in some way or other upon those of the former. We have to find out how. There is a simple way of doing this which is so often used in physics that the student cannot too soon or too carefully learn it. It consists in marking off a series of points whose distances from two fixed straight lines are given by the two series of numbers and then joining these so as to form a continuous line. The two fixed straight lines are called *axes.* They are usually and most conveniently drawn at right angles to one another and are then called *rectangular axes.* For example OX and OY in *Fig.* 40 are two rectangular axes, and for convenience sake are drawn upon squared paper. The horizontal straight line OX is called *the axis of X*, and distances measured along it are called *abscissæ* (a Latin word meaning "pieces cut off"). The vertical straight line OY is called *the axis of Y*, and distances measured along it are called *ordinates.*

If now we mark off along the axis of *X* distances corresponding to our first series of numbers, 10 *mm.*, 20 *mm.*, etc. . . . 100 *mm.*, and along the axis of *Y* distances corresponding to the second series of numbers, in the particular case chosen here 20.8 *mm.*, 41.3 *mm.*, etc. . . . 169.4 *mm.*, we can establish the required system of points as follows :—

By drawing a vertical straight line (or following that already drawn upon the squared paper) through the first mark on the axis OX and a horizontal straight line through the first mark on the axis OY, we can fix a point, *a*, where these cut, whose abscissa, or distance from OY is 10 *mm.*, and whose ordinate, or distance from

OX is 20.8 *mm.*, and by proceeding in the same way with the other pairs of numbers a series of points is obtained. If now these be joined free hand, a continuous line OC is produced. It is the *curve* which shows the relation between the deflections and the masses whose weights at the particular place have produced them.

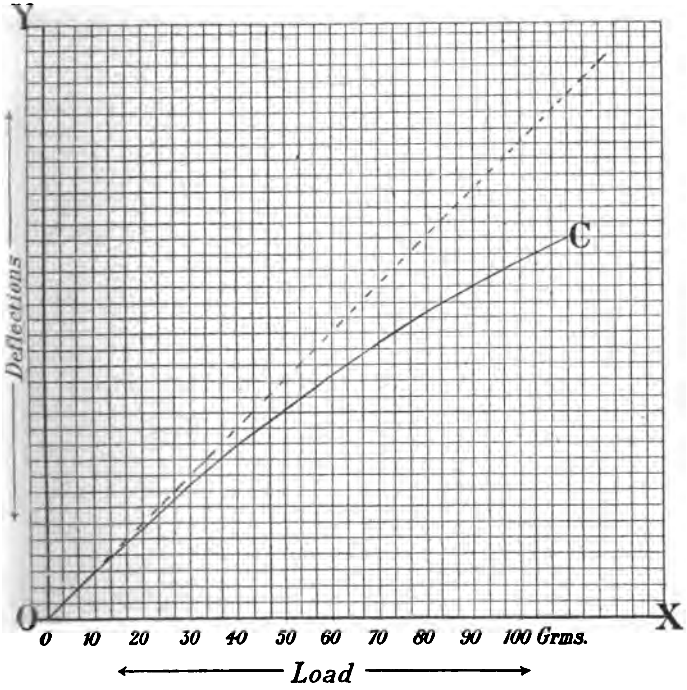

FIG. 40.

Every single point on this curve is at some particular distance from OX, that is, has a certain ordinate, and *is at the corresponding distance from* OY, *that is, has the corresponding abscissa.* Now what we observe with the spring balance are the deflections, which in our diagram are represented by the distances from OX, or ordinates. What we wish to know are the masses whose weights produce these deflections, and these in our diagram are represented by the distances from OY, or abscissæ. We have made the curve by joining points of which both ordinate and abscissa are known. We can now use it to find the abscissa if we are given the ordinate. Thus, if on placing a certain object in the pan we obtained a deflection of say 84 *mm.* (with the balance shown in *Fig.* 36), then we can find the mass of the object by counting off 84 *mm.* along OY, passing along the horizontal line till we reach the curve. Here is the point whose ordinate is 84 *mm.* By passing vertically down from it till we reach the axis OX, we find that its abscissa is 44 *mm.* Then clearly the mass of the object is 44 grammes.

A curve must therefore be drawn, making the abscissæ the number of grammes placed in the pan in Exercise XXIX. and the ordinates the corresponding readings of the pointer.

This curve must be carefully preserved, since it is needed in all cases in order to find the number of grammes corresponding to a particular reading given by the pointer.

When the balance has been some time in use, it must be tested with standard masses occasionally to see whether the readings have altered at all. If they have, a new curve must of course be constructed.

§ 97. DETERMINATION OF WEIGHT BY SPRING BALANCE AND DEDUCTION OF MASS.—The spring balance just described can be used to weigh accurately to 1 gramme, and, by careful estimation, to 0.1 gramme.

Exercise XXX.

To find the mass of an object.

Apparatus.—Spring balance (graduated in Exercise XXIX.).
Objects—*e.g.* new penny, brass ounce weight.

Method.—Place the object in the pan of the balance, "damp" carefully, and read. From the curve obtained in last exercise (§ 96) read off the corresponding mass. (The results will be needed again for comparison.)

Calculate from the observed mass of the ounce "weight" the relation between the pound and the kilogramme.

§ 98. THE LEVER.—It was shown in § 92 that the turning power of a force, when applied to a rigid body supported at one point, might be used as a means of comparing that force with another. A *rigid rod* or bar, supported so as to be *able to turn about one point* is called *a lever*, and the point at which it is supported and about which it can turn is called *the fulcrum.* For example, a poker thrust between the bars of a grate may be considered as a lever, the place where it rests against the bar being the fulcrum.

We have now to examine experimentally the turning powers of various forces when applied to a lever of some kind.

Exercise XXXI.

To construct a form of lever.

Apparatus.—A uniform strip of wood 0.5 *metre* long by about 3 *cms.* wide and 0.5 *cm.* thick, two pieces of brass wire 1.5 *mm.* in diameter and 5 *cms.* long, a piece of steel needle (end of a knitting-needle) about 4 *cms.* long, fine brad-awl (slightly finer than the knitting-needle), a *large* cork, a cork-borer about 1 *cm.* in diameter, sharp knife, two small pieces of straight clock-spring (about 1 *cm.* long), metre rule, cubical block of wood about 6 *cms.* side, Faraday's cement.

Method.—The width of the rod having been measured, a pencil-line is drawn lengthwise along the middle. The length of this line is measured accurately, and three marks are made upon it—one exactly bisecting it, another at one-third of its length, and the last at one-quarter of its length. Three holes are bored through the rod with the brad-awl exactly at these points.

Two other and somewhat larger holes are bored 1 *cm.* above the lower edge and 1 *cm.* from each end.

The pieces of brass wire are bent into the shape shown in *Fig.* 41, and the straight portion is passed through these holes, in which it must fit fairly loosely.

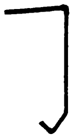
FIG. 41.

The piece of knitting-needle being pushed through the central hole, the rod is ready to be placed upon a suitable support.

This support can be constructed as follows:—

A hole is bored through the cork in the direction of its smaller axis 2 *cms.* from one end. Then the portion shaded in *Fig.* 42 is cut away, a deep groove being thus produced right across the cork. Two cuts are next made in the surface, parallel to and about 0.5 *m.* from the edges of this groove, and into these the two short pieces

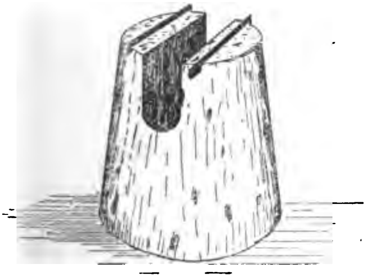
FIG. 42.

of clock-spring are fixed, as in *Fig.* 42, so that their edges project well above the surface of the cork and are in the same horizontal plane.

The complete apparatus will then appear as in *Fig.* 43.

The transverse needle, resting upon the edges of the steel strips, can roll quite freely, and the least force applied to the lever will cause it to turn about its fulcrum.

The bar should now rest exactly horizontal, and, if it does not do so, must be adjusted by filing, or, if necessary, cutting the upper edge at that end which dips down until it does.

We have now to apply known forces to the ends of the bar. This can be done by hanging known masses on to the brass hooks by means of fine cotton.

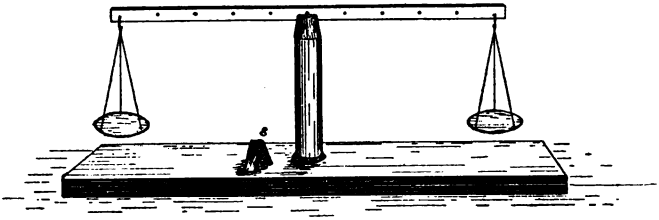

FIG. 43.

Exercise XXXII.

To find the relation between two forces which balance one another when applied to the ends of a lever whose arms are equal.

Apparatus.—Lever (as just constructed) with support in central hole, box of weights, fine cotton.

Method.—Hang a certain mass, say of 20 grammes, on to one hook, and find out what mass must be hung upon the other so as to keep the lever horizontal.

Repeat, using various different masses.

Bearing in mind that the weights of equal masses are equal (at the same place) complete the following statement:—

Two forces, acting upon a lever at equal distances from the fulcrum and so as to turn it in opposite directions, are in equilibrium when . . .

§ 99. LEVER WITH UNEQUAL ARMS.—Having now established the relation between forces that are in equilibrium when acting upon a lever with equal arms, we have to find out what relation exists when the arms are unequal.

To do this, we remove the needle from the central hole and insert it in one of the others. But a difficulty presents itself as soon as the lever is replaced upon the support—it no longer rests horizontal. This is evidently because there is now more of the rod on one side of the support than on the other, and the rod itself being a heavy body the greater weight of the longer arm suffices to turn the lever in that direction. Before we can begin to consider the equilibrium of forces acting at the hooks we must restore the lever itself to equilibrium. This can be done by means of a small leaden saddle, *s* in *Fig.* 43 (a strip of sheet lead), bent so as to fit closely upon the lever (as low down as possible), and placed upon the shorter arm at such a distance from the fulcrum that the lever remains horizontal when there is no load upon the hooks.

The lever as a whole is now heavier than before, but since we are not concerned with the weight of the lever itself at present this will not affect our experiment.

Exercise XXXIII.

To find the relation between forces in equilibrium acting upon a lever with unequal arms.

Apparatus.—As for Exercise XXXII., but with transverse needle in one of the holes other than central, and leaden saddle to restore equilibrium.

Method.—With needle in, say, the third-way hole measure the actual length of the arms from fulcrum to a point in the middle line immediately above the hook. Then hang a known mass upon one hook and find out what mass upon the other is needed to re-establish equilibrium.

Repeat with several different masses and record results thus :—

Series 1.	I.		II.	
	Length of short arm.	*cms.*	Length of long arm.	*cms.*
	Mass on short arm.		Mass on long arm.	
	(*a*)	....		
	(*b*)	....		
	(*c*)	....		

Then change the needle to the quarter-way hole, move the leaden collar as necessary, and repeat experiments, recording lengths of arms and masses whose weights balance as before.

Series 2. I. II. etc.

It will be noticed that the masses needed upon the shorter arm are always greater than those upon the longer.

Taking the numbers obtained in *Series* 1, find out whether there is any constant relation between the forces applied to the shorter arm and those applied to the longer by dividing the numbers of Column II. into the corresponding ones of Column I.

Is there? What is it?

How does it compare with the relation between the lengths of the arms?

See whether any similar relations exist between the numbers obtained in *Series* 2.

Assuming still that the masses hung upon the hooks may be taken as a measure of the forces applied (see § 90), calculate the value of *force* × *length of arm* for Column I. and also for Column II. of *Series* 1. What do you notice?

Repeat for *Series* 2.

It may be inferred from the foregoing results, and from any others that the student may have time to obtain (by boring other holes in the lever and supporting it elsewhere), that the turning power of a force upon a lever depends upon (1) the size of the force and (2) the distance between the line of action of the force and the fulcrum. In our experiments the line of action of the forces applied was vertical (see § 85), and the lever was kept horizontal. Hence we may say that two forces acting upon a lever on opposite sides of the fulcrum, and in the same direction (turning therefore opposite ways), are in equilibrium when the products obtained by multiplying the value of each force by the perpendicular distance between its line of action and the fulcrum are equal.

Now the product of the value of a force and the perpendicular distance between its line of action and a given point is known by a special name; it is *the moment of the force about that point.*

The results of Exercise XXXIII. can therefore be stated shortly as follows:—

Two forces, acting upon a lever so as to turn it opposite ways, are in equilibrium when their moments about the fulcrum are equal.

Hence if the arms be equal, forces that are in equilibrium must also be equal, and this was found to be the case experimentally in Exercise XXXII.

If then two objects, when suspended from the ends of a lever with equal arms, are found not to disturb the equilibrium of that lever, it follows that the moments of their weights about the fulcrum are equal. But, the arms being equal, their weights are therefore equal. Finally, if the weights at the same place are equal, their masses are also equal.

A lever with equal arms thus affords the simplest means of comparing and equating masses. It is what is commonly called *a balance.*

§ 100. THE COMMON BALANCE.—This instrument, as we have just seen, consists essentially of a lever supported exactly in the middle, and so arranged that it turns easily about its support. The particular form used depends upon the purpose for which it is designed. The principle of the rough "scales" used by the shopkeeper is just the same as that of the delicate balance used by the scientific chemist, but there are naturally considerable differences in the details of construction. We shall here describe shortly, not the most delicate kind of balance, but one suitable for simple physical measurements, and capable of affording a comparison of two masses accurate to within 0.01 *gramme.*

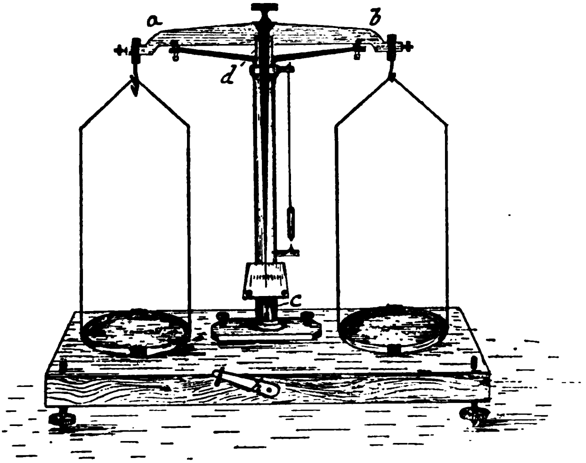

FIG. 44.

Such a balance is shown in *Fig.* 44.

The essential parts are :—

(1) *The beam, ab.* In this case a bar of brass of such a thick-

ness that it does not bend when carrying the largest load which the balance is constructed to bear.

Upon this beam are fixed three knife-edges made of hard steel or of agate. One, exactly in the middle, projects on either side of the beam and takes the place of the transverse needle in our lever of Exercises XXXII. and XXXIII. The other two are near the ends of the beam and point upwards, their purpose being to carry the stirrups from which are presently to be hung the scale pans, and which correspond to the brass hooks in the lever.

The beam also carries a long steel pointer, so fixed as to point vertically downwards when the beam is horizontal. The slightest tilting of the beam produces an easily visible movement of the end of the pointer.

(2) *The support, cd.* This consists of a hollow pillar of brass firmly fixed to the base of the balance, and having a brass rod arranged so as to slide up and down inside it. This rod can be raised or lowered by turning the handle *h* to the right or left respectively. At the top of the rod are two perfectly flat pieces of agate which, when the rod is raised, lift and support the central knife-edge.

Two brass stays project outwards and upwards from the fixed pillar, having at their extremities two V-grooves, into which the beam falls when the agate planes are lowered from beneath the central knife-edge. When the beam is resting in these it is said to be *arrested*, the stays, etc., upon which it rests being called *the arrestment.*

At the foot of the fixed pillar is a small ivory scale over which the lower end of the pointer attached to the beam moves to and fro.

Finally, a plumb-line with a small pointed weight is fixed to the brass pillar, and below it a small point, directed upwards, so placed that when the point of the plumb-bob falls exactly over it the pillar is vertical.

(3) *The base*, which is a slab of some hard wood and rests upon three levelling-screws by adjusting which the central pillar can be made to stand quite vertical.

(4) *The stirrups*, which are to carry *the pans.* These stirrups are square frames of thin brass having agate planes at the top which are hung upon the end knife-edges. Below they have hooks from which the pans are hung.

The agate planes, here and upon the central support, are used because it is essential for the knife-edges to rest upon something very hard and smooth in order that there may be as little friction as possible to hinder the movements of the beam. Steel plates would do, but agate is preferred because it does not rust.

The appearance of the balance when these various parts are duly put together is shown in *Fig.* 44.

It will be noticed that from the ends of the beam small rods project having threads cut upon them and provided with small screws. These are intended to provide a final adjustment. It is not easy to ensure that the arms shall be absolutely uniform, and even if they are, it may happen that the stirrups and pans have not precisely equal masses. To correct any minute inaccuracy these small screws may be moved a little nearer to or a little farther from the centre. As in the case of other fine adjustments already described, the use of a screw enables very small movements to be made accurately and smoothly.

§ 101. USE OF THE BALANCE.—It is first necessary to *be sure that the balance is in adjustment*, that is to say, that the central pillar is precisely vertical and that the beam rests perfectly horizontal when no load is placed upon the pans.

It is best to choose, once for all, a suitable *place in which the balance is always to be used*, and there to make two small conical holes in the bench to receive the two front levelling screws. By this device the danger of moving the whole balance during the operations of weighing (and thus throwing it out of adjustment) is entirely avoided.

The balance having been placed in position and carefully levelled, the pans and beam should be lightly dusted with a camel-hair brush.

Then the handle, *h*, is slowly turned over toward the right so as to raise the beam and pans upon the central knife-edges. Now, what is desired is that the end of the pointer should rest exactly over the middle mark upon the ivory scale, but it is not customary nor right to observe its position while at rest because the various working parts of the balance may have become clogged with dust, and so produce a false appearance of equilibrium. Hence *the pointer is always allowed to swing slowly through a small distance*, and the extreme positions, where it stops for a moment and turns back, are observed, great care being taken to avoid parallax. When these displacements on either side of the middle are equal the pointer would, if still, rest exactly over the central mark. (If the pointer does not swing on raising the beam the balance should be arrested, and the beam slowly raised again until it does.)

Should the pointer now swing farther to one side than to the other the balance is out of adjustment. It must be arrested and the small screw moved slightly in the direction of the longer swing. [Why?] This should very rarely be necessary once the balance has been adjusted. Still it is always necessary, before beginning to weigh, to make sure that the balance is in adjustment.

In arresting the beam always begin to turn the handle back while the pointer is swinging toward the middle point, and finish while it is just over the middle point of the scale, otherwise the beam, falling aslant upon the fixed supports, will be moved bodily along and the balance thrown out of adjustment again.

Exercise XXXIV.

To find the mass of various objects.

Apparatus.—Common balance, box of weights, brass ounce weight, new penny.

Method.—Having dusted the balance and ascertained that it is in adjustment, as just described, put the object to be weighed in the middle of the left-hand pan. Then sit in front of the balance so as to avoid parallax in reading the scale, take the handle in the left hand, and *with the pincers* in the right hand put what appears to be the likeliest weight in the middle of the right-hand pan, and begin very slowly to turn the handle over. The pointer will probably begin to move steadily in one direction or the other, and when it has moved over about four divisions of the scale, the handle is gently turned back again and the balance arrested.

Remember that *the pointer always moves toward the lighter pan.* [Why?]

If the first weight tried be too small remove it and repeat till a weight is found that is too large. Suppose now the 10 gramme weight be found to be too heavy. Remove it and substitute the 5 gramme. Again very gently raise a little way. The pointer now moves, say, to the right. The 5 is too small. Again arrest the beam, and place the 2 gramme weight beside the 5. On raising the beam the pointer still moves steadily to the right, suppose. Arrest, and add the other 2 gramme. This time, it may be, the pointer moves steadily to the left. The weights are too big and the beam must be arrested, one 2 gramme removed and replaced by the 1 gramme. Proceed in exactly the same way with the 0.5, 0.2, 0.2, 0.1, etc.

Never alter this order of placing the weights in the pan, and never add or remove a weight without first arresting the beam.

The arrangement of the weights upon the pan is a matter of some importance, for if care be not taken they may tilt it, which would be inconvenient. The larger weights should be grouped as nearly as possible in the middle, and the smaller ones (from 0.5 to 0.01 gramme) should be placed in two straight rows across the pan in front of the others, as in *Fig.* 45, where weights amounting to 61.63 grammes are arranged in proper order.

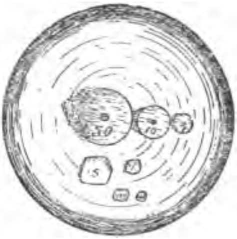

FIG. 45.

Suppose now weights have been added till the pointer swings equally on both sides of the middle. It remains to add up the values. This *must be done by looking* not at the weights upon the pan, but *at the empty spaces in the box.*

A note is made of the value read from the box, and then the weights upon the pan are carefully replaced one by one, beginning with the largest, and a separate count is thus made which serves as a check upon the former. Mistakes are most often made, not in the actual operation of weighing, but in reading off the values of the weights used.

Having found the mass of the penny and that of the ounce weight, compare the results with those obtained with the spring balance in Exercise XXX.

From the mass of the ounce weight calculate the relation between the pound and the kilogramme.

Compare your results with the following, and calculate the percentage error :—

1 *pound*=0.4536 *kilogramme.* 1 *kilogramme*=2.2046 *lbs.*

The student is urged to study carefully the directions just given, since upon the faithful observance of all the precautions indicated depends not only the accuracy and speed of the weighing, but also the preservation in good condition of the balance and the set of weights.

§ 102. THE TWO MEANINGS OF THE WORD "WEIGHT."—It will have been noticed that throughout § 101 the word "weight" is used, without qualification, to describe the standard masses of various sizes contained in the set supplied. This has been done because the word is commonly so used, but the student must bear in mind that this is only one of the many instances in which the popular name of an object is altogether misleading. Each of the pieces of brass or platinum in the box has a certain mass, and because it happens to be upon the surface of the earth, it has a certain weight also. That is to say, the earth attracts and is attracted by it with a certain force called its weight. This weight is not the same at all places on the earth, and would be utterly different were the objects removed to the surface of, say, the moon. What we do with the balance is to find such a mass that the attraction between the earth and it is the same as the attraction between the earth and some other piece of matter. Then we know what is the mass of that other piece of matter. *The balance thus measures mass.*

The spring balance measures weight, for it measures the (shear produced by the) attraction between the earth and the object. By the device of measuring the (shears produced by the) attraction between the earth and various known masses and constructing a curve from the results, we were able to use it to compare masses at one place. But that curve would become useless were the spring-balance removed to some other place, since the weights of the masses used in constructing it would then be different.

When we speak of "weighing" we mean really "measurement of mass."

§ 103. WEIGHING BY DIFFERENCE.—It often happens that the object whose mass is to be determined is of such a kind that it cannot or should not be placed directly upon the scale pan. It is evident that in all such cases the object or substance must be contained in some suitable vessel, and that the mass of substance and vessel together must be determined, as in § 101. The mass of the vessel must be determined separately either before introducing the substance or after emptying it out and subtracted from the mass of vessel and substance together. This method of "weighing by difference" is always used in practical chemistry, and we shall have to employ it later in dealing with liquids (Exercises 51, etc., Chapter IX). See also § 107.

Meanwhile, it should be remembered that only very few substances ought to be placed directly on the pan, since if this become scratched or tarnished the accuracy as well as the value of the balance will be seriously affected.

§ 104. WEIGHING BY SUBSTITUTION.—So far we have assumed that the arms of our balance are of exactly equal length, and upon the correctness of this assumption the accuracy of our results will evidently depend. It may, however, happen that our assumption is not justified, that the arms are only nearly and not exactly equal in length. In that case, as our experiments with the lever have shown us, the masses whose weights are in equilibrium are not equal (see § 99). One very simple way of discovering whether this is or is not the case at once suggests itself. This is, to weigh the object, as in § 101, in the left-hand pan, and then to change it over and weigh it again in the right-hand pan. If the balance is "true," then the apparent mass in either pan will be the same [why? see § 99]; but if "false," it will be greater in one than in the other, and a little thought will show that it will appear greater than the true mass when placed in that pan which hangs from the longer arm and less than the true mass in that pan which hangs from the shorter. [Why? Explain fully.]

But though we can in this way detect the falsity of a balance (and, as we shall see in § 106, calculate the true mass from those observed), it is useful to have a simple device by which any balance, true or false, may be made to give at once an accurate measure of the mass of an object.

This device consists in placing the object to be weighed in one—conveniently the right-hand pan and putting into the other pan a small vessel such as a pill-box. Then small shot are introduced into the pill-box until the addition of one shot renders the pill-box heavier, and its removal renders it lighter than the object. Finally, a piece of tinfoil is held over the pill-box and small pieces are snipped off it with a sharp pair of scissors and allowed to fall into the box until at last it exactly balances the object.

Remember that the beam must be arrested every time shot or tinfoil is added or removed, and that all the other precautions of § 101 must be rigorously observed. The pill-box and its contents are now spoken of as *a counterpoise.*

The object is now removed from the right-hand pan and weights are placed there until the counterpoise is again exactly balanced. The mass of the weights needed (read as in § 101) is exactly the same as the mass of the object.

That this is the case is easily seen if we consider that each in turn has been made, under exactly the same conditions, to counteract by its weight the turning power of the same counterpoise. Consequently the moment of the object and the moment of the weights about the fulcrum must have been the same, and since they were hung from the same arm it follows that their weights at the same place, and hence their masses, must be the same. Thus, even though the balance may have an error (see § 29), we have here a method of weighing which entirely gets rid of it. This is the more important because experience has shown that the error of a balance is not a constant quantity which can be determined once for all, and for which a correction may be added, but that it varies according to the load which is carried in the pans.

§ 105. COPYING THE STANDARDS OF MASS.—Having now a method of weighing which will give results that are quite trustworthy even with an imperfect balance, we may take advantage of it to produce exact copies of any of the standards of mass.

Exercise XXXV.

To produce an exact copy of a mass of 20 grammes.

Apparatus.—Balance, box of weights, pill-box, small shot, tinfoil, sharp scissors, small glass sample-tube, with cork of such a size that it will just go in flush with the top.

Method.—Write "20 grammes" round the cork in ink and leave it to dry. Place the 20 gramme weight in the right-hand pan and the pill-box in the left. Then add shot and afterwards tinfoil to the latter (as in § 104) till it forms an exact counterpoise. Remove the 20 gramme weight, and in its stead place the sample-tube upright upon the pan with the marked cork alongside it, and put shot and then tinfoil into the tube until the counterpoise is again exactly balanced.

Remove the tube and cork and push the latter in till it is flush with the top.

From what was said in § 104, it is clear that the tube with its contents is an exact copy of the 20 gramme mass.

Any of the larger weights may be similarly copied. In case it should be desired to copy the lesser weights, say from 1.0 to 0.01 gramme, it is of course not possible to use glass tubes, shot, etc. In these cases the copy is made either from a piece of brass wire (for the larger 1.0 to 0.2 gramme) or from aluminium wire (for the smaller 0.1 to 0.01 gramme).

Exercise XXXVI.

To make a copy of the 0.5 gramme weight.

Apparatus.—Balance, box of weights, tinfoil, sharp scissors, brass wire, standard wire gauge [S.W.G.] No. 18, cutting pliers, fine file.

Method.—Straighten very carefully a portion of the wire, and measure and cut off exactly 10 *cms.* of it. Weigh this 10 *cms.* of wire to within 0.01 grammes. Its mass will be found to be about 0.91 gramme. Hence each *cm.* of the wire will have a mass of something like 0.095 gramme. Therefore, to get 0.5 gramme of it we shall need $\frac{0.5}{0.095} = 5.3$ (*cms.*) almost. Now cut off as exactly as possible 5.4 *cms.* of the wire. Place the 0.5 gramme weight in the right-hand pan and counterpoise it exactly with pieces of tinfoil placed in the other. Remove the weight and replace it by the piece of brass wire. It will be found that this outweighs the counterpoise, and if it does not it has been taken too short, and a fresh piece must be cut off. It should, however, only just outbalance the tinfoil, or the work of adjusting it will be very tedious.

Having now obtained a piece of wire with a slightly greater mass than the weight we wish to copy, we have next to mark the value it is to have. This can be done by making a series of bends in it and letting each "arm" stand for one "unit" (in this case of course the "unit" is 1.0 gramme). Our piece of wire will therefore be bent into a pentagon, one of the free ends being turned up for convenience in lifting.

It only remains to make the final adjustment of the mass of the bent wire. This is done by rubbing one end of it upon the fine file and from time to time replacing it in the pan until it is found exactly to balance the counterpoise.

It has then evidently the same mass as the weight it replaces.

If it be desired to reproduce any of the other weights the following hints will be found useful:—

For weights 5 grammes to 1 gramme, brass wire S.W.G. No. 12, diameter 0.264 *cm.* Mass of 10 *cms.* about 4.5 grammes.

For weights 0.5 to 0.1 gramme, brass wire S.W.G. No. 18, diameter 0.122 *cm.* (as just used in Exercise XXXVI.).

For weights 0.05 to 0.01 gramme, aluminium wire S.W.G. No. 20, diameter 0.091 *cm.* Mass of 10 *cms.* about 0.17 gramme.

§ 106. FALSE BALANCE.—It was pointed out in § 104 that if the arms of a balance were unequal, the apparent mass of an object would be less than its real mass in one pan and greater than its real mass in the other. It is easy to see how, by observing the apparent mass in each pan in turn, the real mass can be calculated.

From the experiments with the lever, § 99, we learned that two forces acting on a lever are in equilibrium when their moments about the fulcrum are equal, and we defined moment as force × length of arm.

Let us assume that the lengths of the arms of a false balance are l_1 *cms.* and l_2 *cms.* respectively, and that the *real mass* of an object weighed with it is M *grammes.*

The mass which balances it when it is hung from the arm of length l_1 *cms.* is, say, m_1 *grammes.*

Then (taking the masses as a measure of the forces as before) we know that

$$M \times l_1 = m_1 \times l_2 .$$

The mass which balances the object when it is hung from the arm of length l_2 *cms.* is, say, m_2 *grammes.*

Then we know also that

$$M \times l_2 = m_2 \times l_1 .$$

If we multiply these two equations together we get

$$Ml_1 \times Ml_2 = m_1 l_2 \times m_2 l_1 ,$$

or

$$M^2 \times l_1 l_2 = m_1 m_2 \times l_1 l_2 .$$

Dividing both sides by $l_1 l_2$ we find

$$M^2 = m_1 m_2 ,$$

from which

$$M = \sqrt{m_1 m_2} ,$$

i.e. the real mass of an object can be found by taking the square root of the product obtained on multiplying together its apparent masses when weighed successively in each pan of any balance.

Balances are made which have an additional knife-edge at one end placed slightly beyond the proper one, and by moving the stirrup on to this the balance can be made false. It is brought into adjustment by a small counterpoise in the other pan. [Explain why this is necessary.] If such a balance be available the student should use it to prove that the mass calculated as just described is the same as that found on weighing by substitution (*i.e.* is the true mass).

§ 107. AN EASY METHOD OF WEIGHING BY SUBSTITUTION. —The advantages of weighing by substitution are so manifest that it is naturally desirable always to employ this method in preference to that of direct weighing used in § 101. Unfortunately, however, it is a somewhat long and tedious operation to adjust a counterpoise as directed in § 104, and hence for the most part even men of science are apt to sacrifice accuracy to speed by weighing directly. This, however, is by no means necessary, since there is a plain and simple method of weighing by substitution quite rapidly and easily.

It consists in providing once for all several pieces of lead or brass for use as counterpoises. It is convenient to have at least three. One should have a mass of about 50 grammes, another of about 100 grammes, and the third of about 250 grammes. Smaller or larger pieces may also be kept with advantage.

When an object is to be weighed it is placed in the right-hand pan of the balance, and the one of these counterpoises which is as little as possible heavier is put into the other. Then the object is removed, and weights are placed in the right-hand pan till the counterpoise is exactly balanced. Note is taken of the mass required.

The weights are next removed, the object is replaced in the pan, and weights are again added till equilibrium is restored, note being made of the mass needed.

It is clear that the difference between these two masses is the exact mass of the object.

This method is specially useful when a series of weighings has to be made, because then only a single reading of the mass required to balance the counterpoise need be made for the whole series.

The purpose of keeping several counterpoises of different masses is that the balance may be worked with as small a load as is reasonably convenient, for it has been found by experience that the smaller the load the more sensitive is the balance—that is to say, the more easily does it turn when there is a slight difference between the weights of the contents of the pans.

It will be observed that this method combines weighing by substitution with weighing by difference.

§ 108. BALANCE WITH RIDER.—It has been assumed throughout this chapter that the student had access to a balance capable of measuring accurately to 0.01 gramme. Balances far more delicate than this are of course made, and indeed those commonly used in chemical laboratories will weigh accurately to 0.0002 gramme.

These more delicate balances are always enclosed in a case with glass sides, and having a window in front to protect them not only from dust, but also from currents of air, which easily disturb the equilibrium when the beam is swinging. The arrestment is also more elaborate, and provides supports not only for the beam, but also for the stirrups carrying the pans, while the pans themselves are made to rest upon two supports which are raised from the floor of the case by the movement of arresting the beam.

With such a balance weights smaller than 0.01 gramme are of course needed, and these, even when made of aluminium foil or wire, are inconveniently small. Their place is therefore taken by a single small weight of fine wire called *a rider*, which is placed not in the pan, but upon the beam itself at various distances from the fulcrum (see § 98). The rider usually has a mass of 0.01 gramme (centigramme rider). It is carried upon a small hook at the end of a brass rod, which slides along a fixed guide, and is movable from outside the case. The beam is graduated, nine large marks at regular distances from the fulcrum dividing it into ten equal parts. By placing the rider upon one or other of these marks it is possible to add the equivalent of 0.001, 0.002 . . . 0.009 gramme to the weights. These larger divisions are often subdivided by four equidistant marks, and removal of the rider from one of these smaller marks to the next obviously amounts to an alteration of 0.0002 gramme. Such a balance will rarely be necessary for the exercises in this book, and should the student be called upon to use it he must not only observe all the precautions set forth in § 101, but must use it, at first, under the supervision of his teacher.

CHAPTER VIII

DENSITY

§ 109. EXTENSION AND MASS.—We have now considered and seen how to measure two of the most important properties of objects—their extension in space and their mass, or the quantity of matter in them. It is natural to inquire whether there is any connection between them.

In § 105 we saw that, other things being equal, the mass of a piece of wire depends upon its length. Having discovered that 10 *cms.* of No. 18 S.W.G. brass wire had a mass of 0.91 gramme (very nearly), we calculated that 5.4 *cms.* of it would have a mass of just over 0.5 gramme, and found that to be the case. But, clearly, it is not upon the length only that the mass depends, for 10 *cms.* of No. 12 S.W.G. brass wire are found to have a mass of 4.5 grammes. Hence we conclude that it is not extension in one direction only which decides the mass, and we next inquire whether it is the total extension in length, breadth, and thickness—that is to say, the volume.

It was found by actual trial that 46.80 *cms.* of No. 18 S.W.G. brass wire have *the same mass as* 10 *cms.* of No. 12 S.W.G. brass wire (viz. 4.5 grammes).

It is easy to calculate the *volumes* of these pieces of brass wire. Each piece is a cylinder, and we have seen, in § 68, that the volume of a cylinder whose length is l *cms.*, and the radius of whose base is r *cms.*, is $\pi r^2 l$ *c.cs.*

First cylinder. No. 18 S.W.G.
length $= 46.80$ *cms.* $[= l]$
diameter $= 0.122$ *cms.*
$\therefore$ radius $= 0.061$ *cms.* $[= r]$
and $\pi = 3.1416$.

Hence the *volume* = 3.1416 × 0.061 × 0.061 × 46.80 *c.cs.* = 0.5471 *c.cs.*
Second cylinder. No. 12 S.W.G.
length = 10 *cms.*
diameter = 0.264 *cms.*
∴ radius = 0.132 *cms.*
Hence the *volume* = 3.1416 × 0.132 × 0.132 × 10 *c.cs.* = 0.5474 *c.cs.*

Remembering what was said in § 46 about doubtful figures the student will recognise that within the limits of our experiments *these two volumes are the same.*

Hence it appears that equal masses of brass wire have the same volume. Similar measurements would show that equal masses of any one substance have the same volume, and, conversely, that equal volumes of the same substance have the same mass. Now if this be admitted, and it is easily tested for a great variety of substances, the question next suggests itself whether equal volumes of different substances have the same mass. This question is easily answered from everyday experience. A piece of lead and a piece of cork, for example, which have the same volume have very different masses, so different that we can detect the difference in their weights by our muscular sense without the help of the balance. And, conversely, our sense of sight assures us, even without the aid of exact measurements, that a piece of wood having the same mass as a piece of iron has a far greater volume than the latter.

But such rough comparisons, while they assure us that the relation between mass and volume is different for different substances, and so indicate a new property, arising, so to speak, out of the two already considered, are of course not accurate enough for the student of physical science. We shall therefore proceed to make this new property a subject of measurement.

§ 110. DENSITY.—We may express the connection between the mass and the volume of any substance by stating what is *the mass of unit volume* of that substance. If we are told that 1 *c.c.* of cast-iron has a mass of 7.5 *grms.* we can easily calculate the mass of any other number of *c.cs.* Another plan would be to state what is the volume occupied by unit mass of the substance. Thus we might express the above fact equally well by saying that 1 *grm.* of cast-iron has a volume of 0.13 *c.c.*, and from this the volume of any number of grammes could be calculated. Of these two plans the former is almost always adopted in physics; the latter, however, is often used in chemistry.

It is usual to find a single name to express a definite property, and the *mass of unit volume of a substance is called the density of that substance.*

The foregoing statement would then be expressed by saying "the density of cast-iron is 7.5 *grms. per c.c.*"

It will be seen that density is a property in the measurement of which two distinct units have to be used, and hence in specifying densities *both the unit of length and the unit of mass used must be named.* To say, for example, "the density of cast-iron is 7.5" would be sheer nonsense. The number which expresses a density varies with the units chosen. Measured in British units the density of cast-iron is about 468.75 *lb. per cub. ft.*

The measurement of density will thus involve simply measurements of volume (Chapter V.) and of mass (Chapter VII.).

Exercise XXXVII.

To find the density (in *grms.* per *c.c.*) of the various bodies whose volumes were measured in Exercises XXIV. and XXVII.

Apparatus.—Prisms, pyramids, etc., used in Exercises XXIV. and XXVII. (§§ 69 and 72).
Balance, box of weights, counterpoises (§ 107).

Method.—(1) Verify the measurements of volume already made (as directed in §§ 69 and 72).

(2) Find the mass of each object (as directed in § 107).

From the numbers obtained calculate the mass of 1 *c.c.* of each of the objects examined.

Enter results as in following example :—

Description of object, brass cone.

Data for volume determination—

diameter of base = 2.6 *cms.*
vertical height = 4.1 *cms.*

$$\therefore \text{volume} = \tfrac{1}{3}\,(3.1416 \times 1.3 \times 1.3 \times 4.1)\ c.cs.\ \text{(compare § 71)}$$
$$= 7.26\ c.cs.\ \text{(nearly)}$$

mass of cone = 61.63 *grms.*

$$\therefore \text{density of brass (of this cone)} = \frac{61.63}{7.26} = 8.49\ grms.\ \text{per}\ c.c.$$

§ 111.—USE OF DENSITY DETERMINATIONS TO MAKE INDIRECT MEASUREMENTS.—It is easy to see that if the density of a substance and the mass of any object made of it be known the volume of that object can be calculated at once; and similarly, that if the volume be known the mass can be deduced. For example, if we know that the density of cast-iron is 7.5 *grms.* per *c.c.* we can calculate that the volume of an iron casting whose mass is 150 *kilogs.* is 20,000 *c.cs.* (or 20 *litres*). Similarly, if we know that the density of cork is 0.25 *grms.* per *c.c.* we know equally that the mass of 50 *c.cs.* of cork is 12.5 *grms.*

One use of this will be immediately plain. The various rules worked out in Chapter V. for calculating volume from linear measurements are applicable only to objects of certain regular shapes, prisms, pyramids, etc., and we have not so far found a method of measuring the volume of a solid whose shape is irregular. The method indicated above evidently furnishes one way of doing this. If we know of what substance an object is made, and can obtain some other object of regular shape made of the same substance, then by measuring the density of the substance (from the observed mass and volume of the second object as in § 110) and finding the mass of the former object we calculate its volume, however irregular its shape. It is only necessary to be sure that the two are really made of the same substance.

This method, however, is not really so convenient as another which we shall shortly be able to devise.

But there are other applications which are highly important, and should be carefully noticed.

In § 57 we attempted to find the area of an irregularly-shaped card, and obtained an approximate result. [How?] We can now make the measurement much more exactly, as follows:—The card may be regarded as a prism whose height (the thickness of the card) is very small relatively to the area of its base. From the results of § 68 we know that the volume of this piece of card is $a \times h$ *c.cs.* if the area of the face be a *sq. cms.* and the thickness h *cms.*

If now we take a rectangular piece of the same cardboard and measure its length (l *cms.*), breadth (b *cms.*) and thickness (as above h *cms.*) we know that its volume is $l \times b \times h$ *c.cs.*

Now if the mass of this rectangular piece be determined we can find the density of the cardboard. Assuming the mass to be m *grms.* this is

$$\frac{m}{lbh} \text{ grms. per c.c.}$$

The mass of the irregular piece is, say, M *grms.*, and its volume, as we have just seen, is ah *c.cs.*, from which the density is

$$\frac{M}{ah} grms. \text{ per } c.c.$$

If we assume that the two pieces have the same density (which will be the case, or very nearly so) these two quantities are equal, *i.e.*

$$\frac{m}{lbh} = \frac{M}{ah};$$

whence

$$mah = Mlbh.$$

The thickness being the same, we may divide both quantities by h, when we obtain

$$ma = Mlb,$$

or

$$\frac{a}{lb} = \frac{M}{m};$$

or, in words, the area of the second piece bears the same proportion to the area of the former as the mass of the second bears to the mass of the former.

But the area *lb sq. cms.* is known, and the masses are known and hence the unknown area, *a sq. cms.*, can be found.

Exercise XXXVIII.

To find the area of an irregularly-shaped card by indirect measurement.

Apparatus.—Card used in Exercise XXIII. (§ 57).
Another piece of same cardboard, sharp knife, set-square, glass scale, balance, box of weights.

Method.—Carefully cut out a rectangular piece from the spare cardboard and measure its length and breadth exactly. Find the mass of this piece and also that of the irregular piece. Calculate the area of the irregular piece as above, and compare the result with that of Exercise XXIII. (§ 57).

§ 112. FURTHER APPLICATIONS.—Some other applications of great practical importance may here be noticed. One of them has already been used, without stating it, in copying the 0.5 *grm.* weight (§ 105), and has also been stated shortly in § 109. It follows, both from the experiments of Exercise XXXVI. (§ 105), and from those quoted in § 109, that the length and mass of a wire of given diameter are proportional to one another.

For suppose we have two pieces of wire of the same diameter, one of known length (*l cms.*) and the other of unknown length (*L cms.*).

Each piece is, as before pointed out, a cylinder, and, from the results of § 68 the volumes—if the radius be *r cms.*—will be

$$\pi r^2 l \text{ c.cs. and } \pi r^2 L \text{ c.cs.}$$

Suppose the masses to be

$$m \text{ grms. and } M \text{ grms.}$$

The densities are then

$$\frac{m}{\pi r^2 l} \text{ grms per c.c. and } \frac{M}{\pi r^2 L} \text{ grms. per c.c.}$$

If the two pieces of wire be of the same material these are equal:

$$\frac{m}{\pi r^2 l} = \frac{M}{\pi r^2 L},$$

from which

$$\frac{L}{l} = \frac{M}{m},$$

or, in words, the lengths of two pieces of uniform wire are proportional to their masses.

Evidently this relation can be used to find the length of a large coil of wire by determining its mass and comparing it with the mass of a smaller piece of the same wire whose length is known.

A second practical use consists in comparing the densities of two objects in order to decide whether they are or are not composed of the same kind of matter.

For instance a cylinder of copper might be silver-plated, but on finding its density to be, say, 8.9 *grms.* per *c.c.*, we should recognise the imposture if we knew that the density of pure silver is nearly 10.5 *grms.* per *c.c.*

[It is interesting to notice that this application was first made by a famous mathematician, Archimedes of Syracuse, more than two thousand years ago. He devised this plan of finding out whether a crown, made for Hiero, King of Syracuse, was of pure gold or no. (See also § 145)].

Finally, the comparison of the density of a large object with that of a smaller one known to consist of the same material may be used as a means of detecting the presence inside the larger of flaws or cavities, which of course would diminish the apparent density of the material.

Thus if a steel casting were found to have a density much less than 7.8 *grms.* per *c.c.* we should infer the presence in it of a flaw, and could evidently calculate the volume of the cavity.

PROBLEMS :—

(9) The density of pure gold is 19.3 *grms.* per *c.c.* A certain ornament has a mass of 696.6 *grms.*, and its volume is found to be 43 *c.cs.* Does it (assuming it to be solid) consist of pure gold?

(10) The density of a certain specimen of cast brass was 8.1 *grms.* per *c.c.* A casting from it was found to have a volume of 240 *c.cs.* and a mass of 1.92 *kilogrms.* Did it contain a cavity? If so, find the volume of that cavity to the nearest *c.c.*

§ 113. DENSITIES OF CERTAIN SUBSTANCES.—The densities, in *grms.* per *c.c.*, of a few commonly occurring substances are given in the following table. The numbers are approximate only, because in most cases the density varies somewhat with the source of the material and the treatment to which it has been subjected, and, in all cases, is affected by the presence of even minute traces of impurities. Moreover, density varies with temperature. The numbers here given refer to the ordinary temperature of a laboratory (about 15° C.).

Aluminium	2.68	Tin	7.29
Copper	8.87	Zinc	6.86
Carbon (graphite)	2.35	Beeswax	0.96
,, (gas carbon)	2.36	Brass	8.40
,, (charcoal)	1.50	Cork	0.24
Gold	19.36	Glass (crown)	2.60
Iron (wrought)	7.79	,, (flint)	4.50
,, (cast)	7.21	Ice	0.92
,, (steel)	7.82	Paraffin wax	0.87
Lead	11.35	Quartz	2.63
Platinum	21.50	Sand	1.42
Silver	10.47	Wood (pine)	0.56
Sodium	0.97	,, (oak)	0.75
Sulphur (roll)	2.07		

CHAPTER IX

VOLUME, MASS, AND DENSITY OF LIQUIDS

§ 114. FLUIDS.—The objects so far considered have been solids, bodies, that is to say, possessing a shape, regular or irregular, of their own. But, as was mentioned in § 91, and as, indeed, our everyday experience abundantly teaches us, there are many common substances which possess no proper shape at all, which have always to be confined in vessels made of some solid substance, and which then take exactly the shape of this containing vessel so far as they are in contact with it. This fact we can express, as in § 91, by saying that such substances have no power of resisting forces by which it is sought to change their shape, and we have learned that they are called *fluids*.

It is customary, further, to divide fluids into two classes, *liquids* and *gases*. The difference between the two consists chiefly in this, that whereas if a certain quantity of a liquid be introduced into a large vessel it will form a layer on the bottom, a small quantity of a gas introduced into an empty vessel distributes itself uniformly through the whole space offered to it. Another way of stating this difference is to say that a liquid possesses a distinct surface, while a gas only has a distinct surface where it comes in contact with the surface of a solid or a liquid.

With the class of fluids called gases we shall deal in Chapter XI. At present we are concerned with liquids.

§ 115. LIQUIDS.—A liquid, such as water, mercury, olive oil, alcohol, etc. etc., may be shortly defined then as a substance which possesses no proper shape, but is capable of presenting a free surface, and which, therefore, has a definite volume of its own. The inability of liquids to resist change of shape is seen in all their most important and best known properties. We know, for example, that if a vessel full of water be slightly tilted some of the water will flow out of it. But it seems at first sight as if this property were also shared by, at any rate, some solids. If a vessel full of fine dry sand be tilted some of the sand will likewise flow out. Any one, however, who will make the experiment may observe two important differences between the cases. In the first place, the surface of the water remaining in the vessel is quite flat, while that of the sand remaining is inclined. Secondly, the water which flows out spreads itself out into a flat layer as thin as the nature of the vessel into which it is allowed to fall will permit, while the sand which flows out remains more or less heaped up.

Such an experiment proves that while there is a considerable resemblance between a liquid, like water, and a collection of small solid particles, like fine sand, there is also an important difference between them. One of the most striking examples of this resemblance and difference is to be found in the "dunes" or shifting sand-hills which are found on various parts of our own coasts, and of which far larger forms occur in certain parts of South America. These sand-hills flow, so to speak, from place to place as the direction of the wind changes, and on the great plains of South America they flow over and bury houses, railways, etc., covering them with thousands of tons of shifting sand. The surface of these sand-hills is broken up into ridges and furrows exactly like the waves on the surface of some great river. But the difference between this flow and that of water appears at once when we remember that the flow of the sand stops as soon as the wind falls, while the sand waves remain. The river flows on, wind or no wind, but the waves rapidly fall, and leave the surface flat, as soon as the cause which produces them is removed.

What levels the waves on the surface of water is the weight of the raised parts, which, pulling downwards, makes them slide over the parts below, so as to fill up the troughs or hollows. The friction or rubbing of the particles of sand upon one another is great enough to prevent this. The difference between a liquid and a collection of solid particles is also seen in the case of so-called "quicksands." These are large masses of fine sand, the spaces between the particles of which are filled with water. The lessening of the friction between the particles thus produced makes the weight of a comparatively small object able to thrust those particles aside, and so a man, for example, is quickly engulphed. Another similar instance is seen in the "quaking-bogs" of Ireland.

§ 116. VISCOSITY.—While, however, the most characteristic property of liquids is the ease with which different parts slide over one another, it is by no means the case that all liquids are alike in this respect. There are many liquids, the different parts of which move over each other slowly, and with difficulty. For example, ordinary treacle flows very slowly, and when poured remains heaped up for a little while, only presenting a flat surface after some time. Such liquids are said to be viscous, and the property in virtue of which they show this sluggishness of movement is called *viscosity*. Another familiar example of viscous liquid is glycerine.

But although these liquids flow much less readily than the sand of which we have spoken, they differ from it in the fact that they cannot remain heaped up, but sooner or later sink down and present a flat surface. Thus we gain the idea that liquids may be more or less viscous, but that all of them have this in common, that they cannot continue to resist even a small force tending to change their shape, provided it be applied for a long enough time. There are even some substances whose viscosity is so great that they are usually regarded as solids, but which, when tried by the test just named, appear rather to resemble liquids. A common example is pitch. If a flat plate of this apparently solid substance be supported upon three or more small corks, and an object of fair mass, such as a leaden bullet, be placed on the top, then it is found that in a very little while the bullet sinks down into the pitch and the corks push their way up into it. This shows that the pitch cannot resist the long-continued action of small forces; in this case the weight of the bullet on the one hand, and the resistance of the corks to the weight of the cake and bullet on the other.

Although no known liquid is absolutely devoid of viscosity, yet the liquids with which we shall deal in this and the next chapter are so very slightly viscous that we shall not need to take further account of the property here.

§ 117. MEASURES OF VOLUME.—To measure the volumes of liquid substances it is necessary to have vessels which are graduated, that is to say, of which the cubical content up to some particular mark or marks is accurately known. Such graduated vessels are, for exact work, made of glass. To express the volumes contained in them some *unit* of volume must be chosen.

In the metric system the unit is the same as that of § 63, the *cubic centimetre*. The British unit of volume for the measurement of liquids will be discussed below (§ 125).

Of *graduated vessels* for use in physical work four kinds are in common use, and these are divisible into two classes.

The first class consists of *vessels made to contain a stated volume* of water or other liquid. Of these there are two kinds.

(1) *Flasks*, with long narrow necks, provided with ground glass stoppers, and having a mark etched round the neck. When filled up to this mark the flask contains exactly 50 *c.cs.*, 100 *c.cs.*, etc. to 1000 *c.cs.* (1 litre), as the case may be. *Fig.* 46 shows one made to contain 100 *c.cs.*

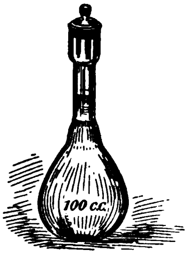

FIG. 46.

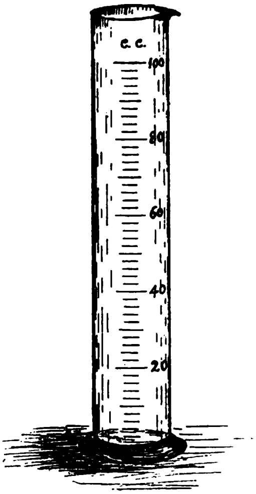

FIG. 47.

(2) *Cylinders*, which are furnished with a series of graduations showing the volumes they contain, from 1 *c.c.* up to the limit of their capacity. *Fig.* 47 shows a measuring cylinder of 100 *c.cs.* capacity with 50 graduation marks, each thus corresponding to 2 *c.cs.*

These vessels, as has been said, are so marked that it needs the specified volume of any liquid to fill them up to the graduation. But it is evident that, if the liquid is one which wets glass, then on pouring out the contents less than the stated volume will be delivered, because some of the liquid remains adhering to the vessel.

Since for various reasons it is often necessary to be able to pour out, or deliver, a stated volume, a second class of vessels is provided, so *graduated as to deliver a stated volume.* Of these again there are two kinds.

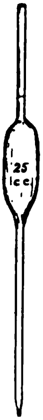

FIG. 48.

(3) *Pipettes*, which correspond to flasks, and deliver a stated round number of *c.cs.* They are glass tubes, usually widened in the middle and drawn off to a fine point at the lower end. These are graduated, so that when filled up to a mark upon the upper stem they will, in emptying, deliver exactly 10 *c.cs.*, 25 *c.cs.*, etc., up to 100 *c.cs.*, as the case may be. *Fig.* 48 shows a 25 *c.c.* pipette.

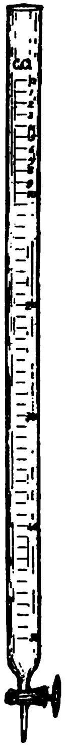

FIG. 49.

(4) *Burettes*, which correspond to cylinders. They are practically long and narrow cylinders drawn out at the bottom and provided with a glass tap. They are so made as to deliver 25 *c.cs.* or 50 *c.cs.*, as the case may be, when the level of the liquid inside is allowed to fall from the topmost to the lowest of the marks upon the barrel. Between these marks is a series of others, each of which corresponds to the delivery of 0.1 *c.c.* *Fig.* 49 shows a burette made to deliver any volume up to 50 *c.cs.*

The use of these vessels evidently depends on observing the exact level of a liquid in them, and before going further the shape of a liquid surface must be observed and considered.

§ 118. SURFACE OF A LIQUID—MENISCUS.—If one of the graduated flasks described in § 117 be filled with water so that the water stands some way up the neck, and the surface be carefully examined, it will be noticed that the statements of § 115, according to which the surface of a liquid at rest is horizontal, appear no longer to hold good when that liquid is confined in a tube. The surface is now seen to be very distinctly curved, forming a sort of cup, with the concave side upwards (*Fig.* 50). The better to examine this phenomenon the following experiments should be made.

Several clean glass tubes of various internal diameters, from about 0.5 *cm.* down to the fine tubing known as capillary, are supported side by side with their lower ends dipping into water. It will be noticed that (1) the surface of the water, originally horizontal, slopes upwards all round the outside of each tube; (2) the surface of the water inside the tubes has a curved shape similar to that of the surface in the neck of the flask; (3) the lowest point of the curved surface inside the tube is above the horizontal level outside; and (4) the rise of the water in the tubes differs in tubes of different diameters.

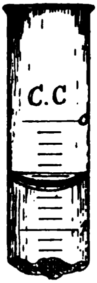

FIG. 50.

A careful examination will show that the greatest rise has occurred in the tube of smallest internal diameter.

[The capillary tube or tubes should be viewed with a magnifying glass.]

If the experiment were repeated with, for example, alcohol, similar results would be obtained, but the actual amount of rise in a particular tube would be found to be different. Now let the tubes, having been carefully dried, be again supported with their lower ends dipping this time into clean mercury. It will be noticed that (1) the surface of the mercury slopes downwards all round the outside of each tube; (2) the mercury inside the tubes has a curved surface, but that this is convex instead of concave upwards; (3) the highest point of the curved surface inside the tube is more or less below the level of the flat mercury surface outside; and (4) the depression of the mercury in the tubes differs in tubes of different diameters.

It will also be seen that this depression is greater in the narrower tubes.

These experiments teach us that the edges of a liquid surface in contact with the walls of a containing vessel have not necessarily the same level as the rest of the free surface, and that, in the case of water in glass, the edges creep up the walls and stand above the ordinary level; while, on the other hand, in the case of mercury in glass, the edges are depressed and stand below the ordinary level.

Further, we see that the free surface of the liquid in a tube is drawn up or depressed as a whole, but less in the middle than round the edges, in the direction of these movements of the sides. This capillary rise or depression, so called because it takes place to an important extent only in very narrow tubes whose diameter is comparable with the thickness of a hair (in Latin, *capillus*), is of great importance and interest; but the only consequence of it which will concern us here is that the free surface of a liquid in a tube is always curved. If the liquid wets the walls of the tube the free surface is concave upwards, and if the liquid does not wet the walls the free surface is convex upwards.

The curved surface in either case is called a *meniscus*.

§ 119. READING LEVEL OF LIQUID.—As a rule we are dealing, in measuring-vessels, with horizontal marks, and have to adjust the height of a column of liquid so that its free surface coincides with one of these marks. But, since the free surface is curved, it is necessary to choose some definite part of it. It is convenient to take the lowest point of a concave or the highest point of a convex meniscus. This is easily done by adjusting the height so that the horizontal mark upon the vessel just touches—*i.e.* forms a tangent to—the curve of the meniscus.

In practice this is carried out by first filling the vessel a very little beyond the required mark, and then slowly withdrawing the liquid till the graduation line forms a tangent to the meniscus.

The method of withdrawing the slight excess of liquid will depend on the nature of the vessel.

With the flask or cylinder it is effected by dipping a fine piece of quill tubing into the liquid, withdrawing it without touching the sides, and jerking out the drop of liquid carried off, the operation being repeated as often as necessary.

With the burette it is only necessary very slightly to open the tap so as to let the liquid drop slowly out. In the case of the pipette, which is filled by suction (see § 120 (5)), the pressure of the finger on the top is slightly relaxed, so that the liquid falls very slowly in the upper stem.

The reading of the level of the meniscus is thus an example of line measurement, and it is by far the most difficult example that commonly occurs, since the lowest (or highest) point of the meniscus is in the middle of the tube, and consequently a considerable distance from the mark outside it. Hence an error due to parallax (§ 13) is particularly likely to creep in, and the greatest precautions must be taken to avoid it.

When, as in the case of the flask or the pipette, the graduation-mark runs right round the neck of the vessel, it is easy by holding the latter up so that the circular mark is seen as a single straight line to ensure that the eye, the mark, and the lower surface of the meniscus shall be in the same plane. But the lower surface of the meniscus is often somewhat shadowy in wide tubes, and hence though this plan works well enough with the pipette it is better in dealing with the flask to adopt that given below.

With the cylinder or burette, where the graduation-mark only extends round part of the circumference, it is more difficult to read accurately. The simplest means of avoiding mistakes entirely consists in taking a strip of *black* paper with one edge cut quite straight, and holding it—or, if preferred, fixing it with an elastic band—so as to form a collar round the tube. It is easy to adjust this collar so that the straight edge, which is placed uppermost, forms a circle in a horizontal plane. By looking across at the light, so that the front and back edges appear to coincide, it is very easy to determine exactly when the lower edge of the meniscus, which will now appear very sharp and distinct, meets the edge of the paper.

§ 120. PRECAUTIONS IN USING MEASURING VESSELS.—The following precautions must always be observed in using measuring vessels.

(1) It should be considered whether the vessel to be used is made to deliver or to contain a specified volume (§ 117).

(2) Vessels must be carefully cleaned before use. The most convenient way of doing this is to introduce a warm strong solution of good yellow soap, together with a few glass beads, and move the vessel so that the beads roll round the sides. The vessel must afterwards be thoroughly washed with soft (if possible with distilled) water.

(3) Vessels, when necessary, should be dried as in § 122.

(4) In using flasks or cylinders they should, when possible, be filled very slightly beyond the graduation-mark and the excess be withdrawn as in § 119.

(5) In using the pipette the point is placed well below the surface of the liquid, and, by sucking gently at the open end, the liquid is drawn up till it stands some distance above the mark on the stem. Then the open end is quickly closed with the finger and the level adjusted by slightly releasing the pressure of the finger, the point meanwhile being kept just in the surface of the liquid. The student must not attempt to measure anything but water until he has learned to do this without the slightest danger of taking any liquid into the mouth.

In discharging the contents of a pipette the latter must be held loosely between the thumb and finger so that it hangs vertically with the point just touching the surface of the liquid as it collects below. A pipette should never be blown through, since allowance is made in graduating it for the drop which remains in the pointed end. It must be noticed also that as the pipette empties some of the contents will remain adhering to the sides, and it is necessary to allow it to drain for a definite time. It is best always to hold it with the point still touching the surface of the liquid while slowly counting five.

(6) In using the burette it must be remembered that two readings of level, both of which are to be carefully recorded, are needed to obtain one volume measurement. The liquid should be run off rapidly till nearly the required amount has been withdrawn, then time should be given for that which adheres to the side to drain down, and the last few drops should be withdrawn very slowly. The drop which collects at the point is part of the volume withdrawn, and must be collected when the meniscus has reached the required mark by raising the vessel in which the outflow is being received and gently touching the point with its inner side.

(7) The position of the meniscus must be read with the special precautions set forth in § 119.

§ 121. MASS OF LIQUIDS.—This must obviously be found by some method of weighing by difference, and it is best to adopt the plan given in § 107.

Two cases may arise, corresponding respectively to measurements of volume contained and volume delivered.

In the former, where it is desired to know the mass of liquid introduced into a vessel, the clean dry vessel (see § 122) is placed in one pan of the balance and a counterpoise of sufficient mass in the other. Weights are added till vessel and weights together balance the counterpoise, and the mass required is recorded. Then the vessel is removed, the liquid introduced, and the process repeated. The difference between the two masses gives that of the liquid introduced.

In the latter, where it is desired to know the mass of liquid that has been poured from a vessel, the vessel and liquid are first weighed, then the liquid is poured out, and the vessel with any liquid that remains weighed again.

§ 122. DENSITY OF LIQUIDS.—We have now seen how to measure the volume and the mass of a given quantity of liquid, and hence can easily find the *density* (mass of unit volume, § 110) of a liquid.

From what has been said above, the easiest way to do this will evidently be to find the mass of a given volume of the liquid. This can be done in either of two ways. We may counterpoise a suitable vessel, such as a small beaker, introduce from a pipette or burette an exactly known volume of the liquid, and determine the increase in mass.

Then knowing the volume and the mass we can calculate the density.

Or we may first determine the mass of a small flask or bottle, graduated to contain a known volume, fill it exactly to the mark and determine the mass again, and then calculate as before. Of these methods the latter is the more usual.

It is carried out either in a small graduated flask like the one shown in *Fig.* 46 (§ 117), or in a special form of bottle (*Fig.* 51) furnished with a long and carefully-ground stopper pierced with a capillary hole. This is so graduated as to hold exactly 25 or 50 *c.cs.*, as the case may be, when completely filled to the very top of the capillary hole..

For reasons which will be explained in § 123 this vessel is usually called a "Specific Gravity Bottle." Meanwhile it must be remembered that its use is to determine densities.

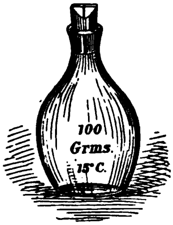

FIG. 51.

The *cleaning and drying* of these small vessels is a matter needing some care.

To clean them they may be washed with a little strong nitric acid, previously warmed in a test-tube.

[N.B.—*Great care must be taken in handling nitric acid. The operation should be carried out in the chemical (not the physical) laboratory, and the acid after use must be poured into a beaker of water before being thrown down the sink.*]

To dry them the vessels, after having been well rinsed with distilled water, are drained as completely as possible, and then warmed by turning them about in the hot air a long way above a Bunsen flame. A current of dry air is then blown through them;

this air being conveniently supplied from a small indiarubber blower like those used with scent-sprays.

[NOTE.—*It will be found much more effective, if the required materials can be procured, to wash out the vessel, after well draining, first with a little alcohol (dried by standing over quick-lime), and afterwards with a small quantity of ether. On blowing air through from the bellows the vessel is dried at once.*]

Exercise XXXIX.

To find the density—in *grms.* per *c.c.*—of various liquids.

Apparatus.—100 *c.c.* flask, "Specific Gravity Bottle" (50 *c.c.*), balance, box of "weights," counterpoises, distilled water, alcohol, olive oil, etc.

Method.—Clean and dry the flask (as directed above). Place the 250 *grm.* counterpoise on one pan of the balance and the dried flask on the other. Add "weights" till the counterpoise is balanced. Take note of the mass required. Then replace the "weights" in the box. Next fill the flask exactly to the graduation-mark with the liquid whose density is to be found, observing carefully the precautions mentioned in § 119. Replace the flask, thus filled, upon the pan, and again add "weights" till the counterpoise is balanced. The difference between this mass and that required with the empty flask is evidently the mass of 100 *c.cs.* of the liquid.

What is the mass of 1 *c.c.?* What is the density? (See § 110.) The correctness of the result clearly depends on the correctness of the weighings and on the assumption that the flask really contains, as stated, 100 *c.cs.* (see further, § 124).

In using the "Specific Gravity Bottle" proceed exactly as already directed to dry and counterpoise the bottle, using in this case a counterpoise of about 100 *grms.*

The filling of the bottle is best carried out as follows:—The bottle is held by the neck between the left forefinger and thumb and filled with the liquid right up to the top. Then the stopper is taken between the right thumb and forefinger and gently lowered into the liquid, with a slight screw-like motion, till it just fits tightly. The outside of the bottle is then wiped quite dry, care being taken, however, *not* to remove the drop which collects at the top of the capillary hole in the stopper.

The bottle is then placed in the pan, and "weights" are added until the counterpoise is nearly, but not quite, balanced. Then, before completing the weighing, the drop remaining upon the top of the stopper is removed by touching it lightly with a piece of clean filter- or blotting-paper. The weighing is then completed as rapidly as possible.

The object of leaving the drop upon the top of the stopper till nearly the end of the weighing is to ensure that the bottle shall be exactly filled at the moment of weighing (compare § 124).

The results give the mass of 50 *c.cs.* of the liquid under examination, and from them the density is easily calculated. As in the former case, the accuracy of the result depends, supposing the weighings to have been properly made, on the bottle's really containing exactly 50 *c.cs.*

Now it will be noticed that, besides a statement of volume, specific gravity bottles—and many other measuring vessels—bear another mark, usually either 60° F. or 15° C. This gives the temperature at which the volume is really that specified, and is important because, as already explained (§ 5), the extension of all objects is altered when they are made hotter or colder. It can be shown that the volume of a vessel is increased when its temperature is raised, and also (as is explained more fully in § 124) that the volume occupied by a given mass of a liquid is greater when it is hot.

Hence it is very important that either the measurements should be made at the temperature stated, or, if they are not, allowances should be made for the changes of volume produced by alteration of temperature. These allowances, however, are difficult to make, and, since it happens that we more often need to know the relation between two densities at the same temperature than to find the absolute value of any one, it is customary to avoid this difficulty by the device described in § 123.

§ 123. UNIT OF DENSITY—SPECIFIC GRAVITY.—This device consists in choosing some one substance whose density is taken as the unit, and then expressing, by means of a number, how many times the density of a particular substance is greater or less than that of the standard. This number is called the *Specific Gravity* of that substance. The substance whose density is chosen as the unity is *water.*

We have seen in Exercise XXXIX. (§ 122) that the density of water is 1 *grm.* per *c.c.*, and from this, by use of the relations given in §§ 66 and 101 (Exercise XXXIV.), it is easy to calculate that in British units this becomes 62.5 *lbs.* per *cub. ft.* (very nearly).

Suppose now we have determined the mass of, say, 50 *c.cs.* of mercury, and found it to be 679.5 *grms.* We know then that the mass of 1 *c.c.* of mercury is $\frac{679.5}{50} = 13.59$ *grms.*—that is to say, the density of mercury is 13.59 *grms.* per *c.c.* This, we could easily calculate, is equal to 849.37 *lbs.* per *cub. ft.*

Then, if we wish to know the specific gravity of mercury, we have only to take its density expressed in any chosen units and divide it by the density of water expressed in the same units.

Thus, in metric units :—

$$Sp.\ Gr.\text{ of mercury} = \frac{13.59\ grms.\text{ per }c.c.}{1\ grm.\text{ per }c.c.} = 13.59.$$

In British units :—

$$Sp.\ Gr.\text{ of mercury} = \frac{849.37\ lbs.\text{ per }cub.\ ft.}{62.5\ lbs.\text{ per }cub.\ ft.} = 13.59.$$

It will be noticed that the specific gravity is merely a *number* which remains the same whatever units of mass and volume may be chosen for the measurement of the densities.

PROBLEMS :—

(11) The density of a specimen of lead is found to be 712.5 *lbs.* per *cub. ft.* What is its specific gravity?

(12) Find the specific gravity of olive oil if 30 *c.cs.* of it have a mass of 27.45 *grms.*

(13) The specific gravity of cast-iron being 7.5 and that of glass 2.5, find the mass of a piece of glass having the same volume as a piece of cast-iron whose mass is 15 *grms.*

It will now be evident that the numbers obtained in the density determinations of §§ 110 and 122 can be used to calculate the specific gravities of the substances there dealt with.

The student will have observed that the number called the specific gravity of a substance is the same as the number expressing its density in *grms.* per *c.c.* He must, however, be careful not to confuse the two on that account. It happens as a consequence of the choice of units in the metric system that the unit volume of water has, under certain conditions (see § 124), unit mass, and hence that the density of water is 1 *grm.* per *c.c.*; but we have only to employ different units in order to see that this identity is apparent only (compare § 125 below). Thus, if the pound and the cubic foot be chosen as units, it is clear, from what has been said above, that the numbers representing the densities change, and that those called the specific gravities do not.

In general, then, if the density of water, expressed in any units whatsoever, be multiplied by the specific gravity of a substance, the product gives the density of that substance in the same units.

§ 124. EFFECT OF TEMPERATURE-CHANGES UPON DENSITY

—MAXIMUM DENSITY OF WATER.—It has been stated already, in § 5, that the extension of a metal bar is altered by making it hotter or colder—that is to say, by altering its temperature. Now this is true for all substances, and in nearly every case the effect of increasing the temperature of an object is to cause it to expand or occupy a larger volume, that of decreasing the temperature to cause it to contract or occupy a smaller volume.

A given object then—say, a brass cylinder—has a slightly greater volume when hot than when cold. Now, so far as our measurements go, there appears to be no difference between the mass of an object when hot and that of the same object when cold. It follows from this that, since the mass remains constant while the volume alters, the density of a substance must be affected by change of temperature.

Remembering the definition of density (§ 110) it will easily be seen that, as a rule, increase of temperature diminishes the density, while decrease of temperature increases the density. [Explain this fully.]

The changes in volume of solids due to changes of temperature are for the most part very slight, and, except in very accurate measurements, are usually neglected.

But these changes are, as was mentioned in § 122, much larger and more important in the case of liquids.

Hence it is not necessarily nor, in fact, always true that the density of water is 1 *grm.* per *c.c.*

It was mentioned in § 94 that the unit of mass in the metric system is the Kilogramme, but it was not stated there that this unit was originally intended to have exactly the same mass as 1 litre (§ 67) of pure water at the temperature designated at 4° on the Centigrade scale (which is the same as 39°.2 on the Fahrenheit scale).

The reason for the choice of this particular temperature lies in the peculiar behaviour of water when heated or cooled, a behaviour which differs from that of all other substances.

It is well known that if water be cooled down sufficiently it changes into a solid substance called ice. If ice be supplied with heat it gradually melts, and finally is entirely converted into water, the temperature remaining the same from the time when the melting begins until it is completed. This temperature is known as 0° C. (or 32° F.).

It is found that during the melting a considerable change of volume occurs, so that, approximately, 11 *c.cs.* of ice at 0° C. yield, when melted, 10 *c.cs.* of water, also at 0°. [If the density of water at 0° C. be ***assumed*** to be 1 *grm.* per *c.c.*, what is that of ice at 0° ?]

If the water at 0° C. be now further supplied with heat its volume should, judging from what is known of other liquids, increase as its temperature rises. But this does not happen. As the temperature rises from 0° C. to 1° C. the volume of the water slightly but certainly diminishes, and continues to do so from 1° C. to 2° C., 2° C. to 3° C., and 3° C. to 4° C. At this temperature comes a change. As the water is heated from 4° C. to 5° C. it no longer contracts, but begins slightly to expand, and continues to do so until it boils. Between 8° C. and 9° C. it regains the same volume as it had at 0° C.

The point which concerns us here is, that if water be warmed from 0° C. up to 4° C., or cooled from any higher temperature down to 4° C., it steadily contracts. A given mass of water has a smaller volume (and therefore a greater density) at 4° C. than at any other temperature.

Thus 4° C. *is the temperature of maximum density of water.*

The student will prove this important fact experimentally when he comes to the study of Heat. Meanwhile it must be borne carefully in mind.

§ 125. RELATION BETWEEN UNITS OF MASS AND UNITS OF VOLUME.—The definition of the kilogramme as having a mass equal to that of 1 cubic decimetre of water at its temperature of maximum density, evidently establishes a simple relation between the units of mass and volume (and hence of length [explain why]).

As already hinted later measurements have shown that this mass was not copied quite exactly, and the standard kilogramme has a mass smaller than that of 1 cubic decimetre of water at 4° by an exceedingly minute amount. To all intents and purposes this difference is unimportant, being only about thirteen thousandths of a milligramme.

In the British system of weights and measures the standard of *mass* is, as explained in § 94, the *pound* avoirdupois. The unit of *volume* in common use is not the cubic foot, but the *gallon.* This volume is equal to 277.274 *cub. ins.*, and is defined as that occupied by 10 *lbs. of water at* 62° *F.* The gallon is divided, as is well known, into 4 quarts, each of which has thus a volume of 69.318 *cub. ins.*, and a quart is divided into two pints (each of 34.659 *cub. ins.*). It will thus be seen that there is no simple relation between the measures of mass and length in the British system.

§ 126. PYKNOMETER.—From what has been said it is evident that in determining the densities of liquids it is of the utmost importance that the temperature at which the determination is made should be accurately known.

This condition might be fulfilled by standing the flask or specific gravity bottle used in Exercise XXIX. § 122 in a bath of water or other suitable liquid, kept constantly stirred and maintained at a definite temperature. By allowing the vessel, slightly overfilled, to stand in such a bath for a considerable time, and making the final adjustments after the contained liquid has attained the temperature of the bath, very good results may be obtained.

But the manipulation is difficult, and it is more convenient to use a special form of vessel known as a **pyknometer.**

[*Note.*—This name is derived from two Greek words, meaning "compact" and "measure" respectively. Hence a means of measuring density.]

A convenient form of pyknometer is shown in *Fig.* 52. It consists of a U-tube, the ends of which are narrowed to thick-walled capillary tubes, bent as shown in the figure so that it can be supported upon them with the body of the tube hanging inside a tall beaker. One of the ends, *p*, is drawn out to a fine point, the other is provided with a mark, *m*, and—beyond this—a series of small bulbs blown on the upper side only of the tube (see *Fig.* 52 (*a*)).

In using the pyknometer, it is cleaned and dried (see § 122 [note]), and then hung by a piece of thin wire upon the hook above one pan of the balance. A counterpoise, of 100 *grms.* or so, is placed in the other pan, and "weights" are added to the pan below the pyknometer till equilibrium is established.

Then the pyknometer is detached from the wire and filled as follows :—

A piece of narrow indiarubber tubing is slipped over the wide end *q*, the point *p* is immersed in the liquid under examination, and, by sucking gently at the open end of the indiarubber tube, liquid is drawn in. It will be found that the side of the tube nearer to the end *q* is the first to be completely filled, a large bubble of air being left enclosed in the other. This must be expelled by blowing gently down the tube (which is held in the position shown in the figure) until the liquid just emerges from the point *p*. By pinching the indiarubber tube it can be held at this level while *p* is re-immersed. Then the suction is continued until the vessel is filled somewhat beyond the mark *m*.

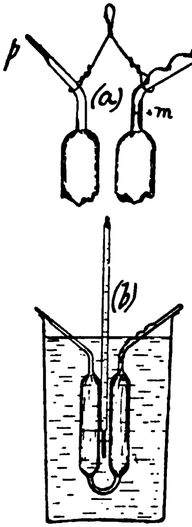

FIG. 52.

The pyknometer, thus somewhat over-filled, is then lowered into the beaker which contains water of known temperature and allowed to remain there for at least a quarter of an hour, the water in the beaker being meanwhile very gently stirred. The temperature is noted on a thermometer immersed in the bath.

Then the level of the liquid is adjusted as follows :—

The end *q* is raised gradually until the end *p* is as nearly as possible horizontal. This will bring the mark *m* into such a position that the level of the liquid in *q* can be read as easily as that in the pipette (§ 117). Then the point *a* is lightly touched with a small piece of filter- or blotting-paper. It will be found that this absorbs a small quantity of liquid with which it comes in contact, and the level of the liquid in *d* will fall. With a little practice it is easy to adjust it so that the surface coincides exactly with the mark *c*.

The pyknometer is now exactly filled with liquid at the required temperature.

On lowering the end *q* into its original position the liquid will recede some way from the point *p*, rising a corresponding amount in the limb *q*. Thus any danger of loss by accidentally touching *p* during the subsequent manipulation is avoided, and, as the only communication with the air is through two capillary holes, any loss by evaporation is made very slight. One use of the bulbs on the limb *q* will now be evident: they permit the storage of this back-flow without filling the tube to an inconvenient height, and by their larger volume cause a proportionately greater fall in p for a given rise in q.

Finally, the pyknometer is removed from the bath and the outside of it thoroughly wiped with a clean dry cloth (great care being taken that the liquid is not allowed to flow back to the point p), and it is again hung in the balance with the same piece of wire.

[*Note.*—To avoid possible mistakes, the suspending-wire should be left in the balance-case during the filling.]

It is left hanging for five minutes or so to acquire the temperature of the air, and then the mass needed to balance the counterpoise is again observed.

The difference between the two masses is clearly the mass of liquid, which exactly fills the pyknometer at the temperature of experiment.

But to deduce the density from this result we should require to know the volume of the pyknometer, and this will manifestly, from what has been already said, vary with the temperature at which it is used.

It is possible, as the student will learn when he comes to the study of Heat, to find out how the volume of a glass vessel varies when its temperature is altered, but we shall here adopt a plan which avoids the somewhat tedious calculations necessary for such determinations.

The density of pure water at various temperatures has been very accurately determined by various investigators, and in § 127 a short table of results from 0° C. to 30° C. is given.

Now it is clear that the pyknometer affords us an easy and accurate means of comparing the density of any liquid at a particular temperature with that of water at the same temperature, for we have only to repeat the operations described above, using water to fill the pyknometer, in order to find the mass of a quantity of water whose volume is exactly equal, at the same temperature, to that of the quantity of liquid whose mass we have already determined. We do not know, it is true, what this volume is, but we may suppose it to be, say, v *c.cs.*

Then, if the mass of the liquid which fills the pyknometer at a particular temperature be, say, m *grms.*, and that of the water which fills it at the same temperature, say, w *grms*, we have—

$$\text{Density of liquid} = \frac{m}{v} \textit{ grms. per c.c.}$$

$$\text{Density of water} = \frac{w}{v} \textit{ grms. per c.c.}$$

and therefore (§ 123)

$$\text{Specific gravity of liquid} = \frac{\frac{m}{v}}{\frac{w}{v}} = \frac{m}{w} =, \text{ say, } s.$$

From what was said in the concluding paragraph of § 123 it follows that if the density of water at the temperature of experiment be D *grms.* per *c.c.*, and the specific gravity of the liquid found as above be s, then—

the density of that liquid $= D \times s$ *grms.* per *c.c.* at the temperature of experiment.

§ 127. DENSITIES AND VOLUMES OF WATER AT THE VARIOUS TEMPERATURES.—In the following table column I. gives the density of water, in *grms.* per *c.c.*, at temperatures from 0° to 30° C. Column II. gives the volume, in *c,cs.*, occupied by 1 gramme of water at these temperatures specified :—

Temperature.	I. Density, in *grms.* per *c.c.*	II. Volume of 1 *grm.*
0° C.	0.999871	1.000129 *c.cs.*
1°	0.999928	1.000072
2°	0.999969	1.000031
3°	0.999991	1.000009
4°	*1.000000*	*1.000000*
5°	0.999990	1.000010
6°	0.999970	1.000030
7°	0.999933	1.000067
8°	0.999886	1.000114
9°	0.999824	1.000176
10°	0.999747	1.000253
11°	0.999655	1.000345
12°	0.999549	1.000451
13°	0.999430	1.000570
14°	0.999299	1.000701
15°	0.999160	1.000841

	I.	II.
Temperature.	Density, in *grms.* per *c.c.*	Volume of 1 *grm.*
16° C.	0.999002	1.000999 *c.cs.*
17°	0.998841	1.001160
18°	0.998645	1.001348
19°	0.998460	1.001542
20°	0.998259	1.001744
21°	0.998047	1.001957
22°	0.997828	1.002177
23°	0.997601	1.002405
24°	0.997367	1.002641
25°	0.997120	1.002888
26°	0.996866	1.003144
27°	0.996603	1.003408
28°	0.996331	1.003682
29°	0.996051	1.003965
30°	0.99577	1.00425

These figures show plainly how considerable is the variation in the volume of a given mass of water when its temperature is altered.

Thus 1 litre of water, which at 4° C. has a mass of almost exactly 1000 *grms.*, has at 15° C. (about ordinary temperature) a mass of only 999.16 *grms.*, a difference of 0.84 *grm.*, an amount easily detected. Even in dealing with 100 *c.cs.*, a difference of 0.084 *grm.* would be found, an amount too large to be neglected in accurate working.

Similarly, if we were, as is obviously possible, to calculate the volume of a given quantity of water from its observed mass (compare § 111), we should introduce a considerable error were we to neglect the circumstance that the volume of a kilogramme of water, which is 1000 *c.cs.* at 4° C., becomes 1000.84 at 15° C.

The utility of the numbers in column I. has already been indicated in § 126, and that of the numbers in column II. will be pointed out in §§ 129 and 130.

§ 128. APPLICATION.—If a balance with rider (§ 108) be available the student should now determine accurately the specific gravity of some liquid or liquids at a chosen temperature (say that of the tap water in the laboratory), and from this, and the density of water at that temperature as given in the table of § 127 (column I.), calculate the density of the liquid or liquids.

Exercise XL.

To find the true density, in *grms.* per *c.c.*, of a liquid or liquids.

Apparatus.—Pyknometer (§ 126, *Fig.* 52), balance, box of "weights," fine wire, counterpoises, alcohol or brine, etc.

Method.—Proceed, as directed in § 126, to find the masses of water and of the liquid which exactly fill the pyknometer at the particular temperature. Then calculate the specific gravity (how? see § 126) and multiply the density of water at this temperature by the number so found.

[*Note.*—The reason why the use of a delicate balance is recommended for this exercise will be manifest. A pyknometer, as a rule, holds something less than 50 *c.cs.*, and hence the difference between the mass of the water which fills it at 4° C., and that which fills it at, say, 15° C., will be less than 0.04 gramme, a quantity so small that any error in it will be of considerable importance.]

§ 129. CALIBRATION OF MEASURING VESSELS.—It will be readily understood that the numbers of column II., § 127, afford an easy means of checking, and, if need be, correcting the statements of volume of graduated vessels. Thus with the "containing" vessels, flasks, or cylinders (§ 117) it is only necessary to determine the mass of water which at a known temperature exactly fills the vessel to the particular graduation-mark, and then, knowing from the table the volume oocupied by 1 *grm.* of water at that temperature, to calculate the volume up to that mark. With "delivering" vessels, the pipette or burette (§ 117) it is necessary to determine the mass of water delivered either on emptying the vessel completely (as with the pipette) or on withdrawing the liquid between two stated marks (as with the burette), and to calculate the volume as before. It is thus possible, and for refined measurements, such as have to be made in chemistry, absolutely essential, to determine the real volume delivered between each *c.c.* graduation upon a burette.

[*Note.*—In thus "calibrating" a pipette or burette it is essential to receive the liquid run out into a tared vessel provided with a stopper which has been weighed with the vessel so as to avoid loss by evaporation during the weighing.]

§ 130. INDIRECT MEASUREMENT OF THE VOLUME OF A SOLID.—It is a matter of common experience that if a vessel be completely filled with a liquid, and a solid object be then immersed in it, some of that liquid will, as we usually say, run over. This is a familiar instance of the rule that space occupied by one object is not available for occupation by another object. There are, it is true, apparent exceptions to this rule. A vessel "filled" with dry sand may have a considerable quantity of water or other liquid poured into it without any of the contents overflowing, and a number of objects, such as a sponge, a piece of unglazed earthenware, etc., will—to use a common expression — absorb considerable quantities of liquids. But a little consideration will show that these exceptions are only apparent. The vessel is not really "filled" with the sand at all. There are spaces between the grains of sand, and in these the water, or other liquid, collects. Indeed, if we reverse the process imagined above and add sand to a vessel previously filled with water, it is found that some of the water overflows. A little reflection and a few experiments will show that such cases are all capable of being accounted for by the existence of cavities or "pores" in the apparently solid material, and we speak of such materials as *porous*.

Admitting then that a solid when immersed in a liquid displaces some of that liquid, we are led naturally to ask the question : What relation exists between the volume of an immersed solid and the volume of liquid displaced by it ?

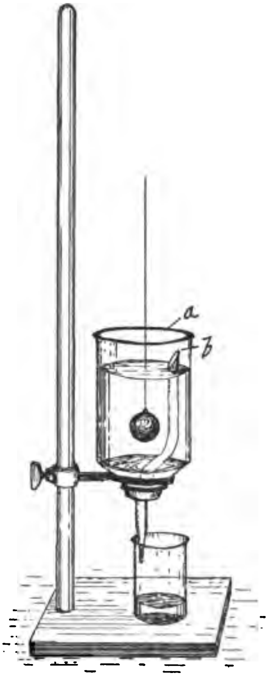

FIG. 53.

One obvious method of answering this question will consist in collecting the liquid displaced by a solid whose volume is already known and determining the volume of this liquid. A simple form of apparatus for this purpose is shown in *Fig.* 53. It consists of a vessel, *a*, which may be an ordinary bell-jar, or may be made by cutting off the bottom of a large bottle with a good file, kept moist with camphor and turpentine. The neck is provided with a well-fitting cork, through which passes a glass tube, *bc*, about 4 *mms.* in internal diameter. The upper end, *b*, of this tube stands 3 or 4 *cms.* below the edge of the bell-jar, and is ground into the bevelled shape shown in the *Fig.* so as to have a clean edge. The lower end, *c*, is drawn out so as exactly to resemble the delivery jet of a pipette. A thermometer is supported so that its bulb is within the vessel and about half-way between *b* and the cork. Water is poured into the jar until it begins to run down *bc*, the overflow being caught in a small beaker, and the tube being allowed to drain for a definite time with the point *c* just touching the water in this beaker (as in discharging a pipette—see § 120, 5). If now a clean dry beaker be placed below *c*, and a solid whose volume is already known (from linear measurements—see Exercises XXIV. and XXVI. §§ 69 and 71) be suspended by a fine thread and lowered steadily into the liquid till it is completely immersed, it will be easy to collect the overflow, draining the tube for the same time as before, and, having determined its mass in the usual way, to calculate its volume from the data given in column II. of § 127.

Exercise XLI.

To determine the volume of water displaced by a solid of known volume, which sinks in water, when it is completely immersed.

Apparatus.—Objects whose volumes were determined in Exercises XXIV. and XXVI., bell-jar, cork, cork borers, rat-tail file, glass tube bevelled at one end and drawn out at the other (as in *Fig.* 53), thermometer, beakers, balance, box of weights, counterpoises, fine thread.

Method.—Fit up the apparatus described above and shown in *Fig.* 53. A *cork* should be chosen very slightly too large for the neck of the bell-jar. If this be wrapped in paper and well rolled under the foot it will become soft, and fit easily, but tightly. In *boring a cork* choose a sharp borer, very slightly smaller than the tube which is to be inserted, stand the cork upright upon the bench and bore with half-turns backward and forward as nearly as possible half-way through it, taking great care to keep cork and borer vertical all the time. Then withdraw the borer, carefully mark with a pencil the position upon the other end, which is exactly opposite to the boring already begun, and from this point bore, as before, till the two holes meet in the middle of the cork. Push out the core, and clean the edges of the hole by laying the cork on its side, passing the rat-tail file through it and rolling the cork to and fro while lightly drawing the file backwards and forwards. Before attempting to insert the glass tube see that the hole is perfectly clean and straight. Wet the walls of the hole and the outside of the tube with water, and push in the tube with a screwing motion, grasping it, not at the extreme end, but with the right thumb and forefinger as near to the cork as possible.

Determine the volume of the overflow when various solids of known volume are immersed, adopting the method and observing the precautions mentioned above.

What relation can you establish between the volume of an immersed solid and the volume of liquid displaced by it?

[*Note.*—The student is warned that this method is not perfectly accurate. Indeed, the results cannot be trusted beyond the first place of decimals, though with care the second place may be made fairly near the truth. A better and more accurate means of finding the volume of a solid is worked out in § 142.]

The student will now have no difficulty in seeing how the result arrived at in Exercise XLI. will enable him to solve the simple problem presented in Exercise XLII.

Exercise XLII.

To determine the volume and density of an irregularly-shaped object, for example a large pebble.

Apparatus.—That used in Exercise XLI.

Method.—To be devised by the student.

Another method, not less obvious, of finding the volume of any solid consists in determining the mass of a liquid of known density which will exactly fill a given vessel, *e.g.* a specific gravity bottle, and then finding what mass of the same liquid is required to fill the vessel after introducing a solid of any form. Thus, for example, we may determine, as in Exercise XXXIX. (§ 122), the mass of distilled water which exactly fills our specific gravity bottle, and then, having dried the bottle, insert a number of lead shot or some washed sea-sand or some brass filings, and proceed to balance the bottle, together with the solid thus introduced, against a suitable counterpoise. If now the bottle be carefully filled with water and again balanced against the same counterpoise the difference between the masses needed will give the mass of water which now suffices to fill it. The difference between this mass of water and that which is needed to fill the empty bottle, evidently gives the mass of water which is displaced by the solid introduced, and from our knowledge of the density of water we can calculate the volume of this, and hence of the solid introduced. Provided we have also determined the mass of the solid introduced we can thus find its density.

Exercise XLIII.

To find the density of a solid which is in small pieces.

Apparatus.—Specific gravity bottle, balance, box of "weights," counterpoise, lead shot or brass filings, etc., distilled water.

Method.—Find the masses needed to balance against the counterpoise—(1) the dry sp. gr. bottle; (2) the dry sp. gr. bottle with shot or filings, etc.; (3) the sp. gr. bottle with shot, etc., filled with water; (4) the sp. gr. bottle filled with water.

The difference (2) – (1) gives mass of shot or filings, etc.

(3) – (1) gives mass of water which fills bottle in presence of shot.

(4) – (1) gives mass of water which fills bottle.

Calculate the volume, and hence density, as above.

§ 131. MEASUREMENT OF THE DIAMETER OF A CAPILLARY TUBE.—Another problem easily solved by taking advantage of the knowledge of densities of liquids is that of determining the diameter of a very fine, or capillary, tube. The bore of such a tube is, or is intended to be, a cylinder whose length is equal to that of the tube, say l *cms.*, and whose base has an area of πr^2 *sq. cms.*, if r *cms.* be the radius (or $2r$ *cms.* the diameter) of the bore.

The volume of the cylinder is thus (§ 68) $\pi r^2 l$ *c.cs.*

Now if such a tube be filled with a liquid whose density is known, and the mass of that liquid which it can contain be determined, it is clearly quite easy to find this volume. For suppose the tube contained m *grms.* of a liquid whose density was known to be D *grms.* per *c.c.*, then it is manifest that the volume of the bore is $\frac{m}{D}$ *c.cs.*

But this we know to be equal to $\pi r^2 l$ *c.cs.*

Now we can make l as long as we please, and it is easy to choose such a length that its measurement need involve no serious error.

Hence we have

$$\frac{m}{D} = \pi r^2 l.$$

Whence

$$r^2 = \frac{m}{D\pi l} = \text{, say, } A \textit{ sq. cms.}$$

$$\therefore\ r = \sqrt{A} \textit{ cms.}$$

Exercise XLIV.

To find the diameter of capillary tubes.

Apparatus.—Clean dry capillary tubing in lengths of 15 to 20 *cms.*, clean dry mercury, glass scale, small beaker, balance, box of "weights," small counterpoise.

Method.—A thread of mercury is drawn up into the capillary tube by sucking at the end of a piece of indiarubber tubing slipped over one end, the tube is turned into a horizontal position, and the indiarubber tubing slipped off. The thread of mercury, which should not more than two-thirds fill the tube, is brought so that one end is about 2 cms. from one end of the tube, and its length is carefully noted. It is then, by slightly inclining the tube, moved a little farther along, and its length again noted. Three or four readings of length should thus be taken, estimating to the nearest half *mm.*, and the average of them recorded as the true length. The reason for taking these several measurements is not only that we may avoid accidental errors of reading (see § 17), but also that we may thus allow for slight inequalities in the diameter of the bore.

The length of the column of mercury having been thus determined, it remains to find its mass. For this purpose the small beaker, carefully cleaned and dried, is balanced, by means of the required "weights," against the counterpoise. Then (the beaker being removed from the balance case) the thread of mercury is carefully tilted into it, and the "weights" needed to balance it against the counterpoise again determined (cf. § 107). By repeating these operations with the same or other pieces of capillary tube, the required data may be collected. It will be noticed that it is not necessary to balance the beaker and its contents afresh before each addition of mercury, the second reading of one experiment supplying naturally the former of the succeeding experiment.

The data having been found, the radii (and hence the diameters) are calculated as above.

[*Note.*—In using mercury the student must bear in mind that it is very costly. All operations with it should be conducted upon a clean wooden tray with raised edges. Further, it must not be forgotten that mercury attacks and dissolves most common metals, except iron, and hence great care must be taken not to let it touch any part of the balance, "weights," etc.

Mercury may be cleaned, when dusty, by filtering it through a cone of blotting-paper, in which a small hole has been pricked with a needle. See also Exercise LXII. *Note*, § 152.]

§ 132. TABLE OF DENSITIES OF CERTAIN LIQUIDS.—The following are approximate values for the densities of some commonly occurring liquids. Since, as we have seen, the density of a liquid is greatly affected by change of temperature, it is important to bear in mind that the numbers here given refer to the temperature of 0° C., and express the mass in *grms.* of 1 *c.c.* of the liquid of that temperature.

Alcohol (absolute)	0.79
Alcohol (methylated)	0.83
Benzene	0.90
Carbon disulphide	1.29
Ether	0.74
Mercury	13.60
Milk	1.03
Olive oil	0.92
Turpentine	0.87
Sulphuric acid	1.84
Water	0.999 (see § 127)

CHAPTER X

SOME OTHER PROPERTIES OF LIQUIDS

§ 133. DOWNWARD THRUST OF A COLUMN OF LIQUID.—We have learned that liquids possess mass, and consequently (§ 90) weight. Now the weight of a given quantity of any liquid is evidently borne by the vessel which contains it. The material of which this vessel is made being more or less rigid, a stress is opposed to the strain (§ 91) produced by the weight of the contained liquid. The rigidity of glass, and most other materials in common use, is so great that we cannot, as a rule, measure this strain.

If we desire to know the force with which a liquid presses against any part—say, the base—of its containing-vessel, we must have a vessel in which that part is made of some material that will show a measurable strain or alteration of shape. Such a vessel may be made as follows :—

An ordinary thistle-funnel, about 45 *cms.* long, has the edge of the bulb ground flat upon a piece of plate glass with emery-powder (§ 11) or with carborundum. It is then thoroughly cleaned and dried. A piece of new, thin, sheet indiarubber, such as is used for mending pneumatic tyres, is cut out, large enough almost to envelop the bulb of the thistle-funnel. The ground edge and the outside of the rim of the latter are then smeared with ordinary rubber solution, and the sheet-rubber, of which the inner face has been cleaned with fine glass-paper and similarly treated with a thin layer of rubber solution, is laid flat over the opening. Then, with very *slight* stretching, it is folded round the bulb and held in this position with the left hand, while it is bound firmly to the constricted portion near the edge with thin copper binding-wire. The whole is left for at least an hour that the solution may dry thoroughly. The completed tube is depicted in *Fig.* 54.

We have now to examine the effect upon this partially rigid base of introducing a liquid, say water, into the open end of the tube. This must be done from the fine jet of an ordinary wash-bottle (such as is used in the chemical laboratory), the jet being held against the inside of the tube.

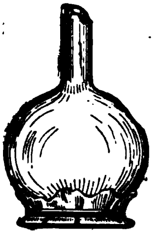

FIG. 54.

First hold up the thistle-funnel with the bulb downwards, and look across the surface of the rubber base: notice that it is flat.

Now introduce water until it stands a few *cms.* high in the narrow tube. Notice that the rubber bulges outwards a very little.

Add more water, and notice that the rubber protrudes slightly more.

§ 134. PROPORTION BETWEEN DOWNWARD THRUST AND DEPTH.—We have now to try to get a measure of this strain which will afford us a measure of the force which produces it (compare § 91).

This can be done as follows :—

A strip of wood, *ab*, about 30 *cms.* long, 1 *cm.* wide, and 0.2 *cms.* thick is supported about 5 *cms.* from the end *a* by a transverse needle *c*. This needle rests in two V-grooves filed in two short pieces of clock-spring set in transverse slots at the top of a vertical wooden pillar, which also has a deep groove across it to receive the rod *ab*. (*Fig.* 56).

The end *b* of the rod is sharpened to a fine point. About 1 *cm.* from the end *a* a small hole is bored in which an ordinary needle will fit tightly—*d* in *Fig.* 56.

Then a pan is constructed as follows :—

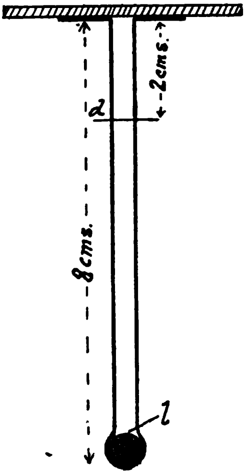

FIG. 55.

A strip of tin-plate, about 1 *cm.* wide and 20 *cms.* long, is bent by means of pliers so as to form a T-shaped piece with the two long limbs about 0.4 *cms.* apart, and the cross-piece about 1.5 *cms.* long In the long limb about 2 *cms.* below the cross-piece, two small clean holes are punched, very slightly larger than the needle *d* (*Fig.* 55), and exactly opposite one another. The lower ends of the long limbs are bent round so as to form a loop upon which a small piece of lead, *l* (*Fig.* 55) may be hung. The appearance which the tin-plate should present when bent is shown in *Fig.* 55. Upon the top of this T-piece a small circular disc of tin-plate, or, conveniently, a penny, is fixed

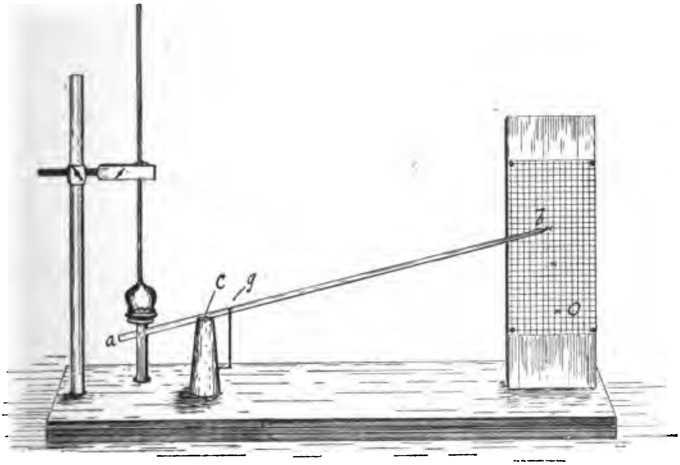

FIG. 56.

with solder or with ordinary cement so that the long limbs are as nearly as possible under its centre.

The whole apparatus may then be set up thus :—

The scale pan is slipped on over the end *a* till the two holes come opposite the hole in the rod, and the needle *d* (*Fig.* 55) is pushed home till firmly fixed in the wood. If the loop at the lower end be now loaded with a small piece of lead the pan will, even when loaded with weights (as in Exercise XLV. below), remain horizontal in all positions of the rod.

The rod is now placed with the fixed needle, *c*, in the V-grooves of the support, and brought into an approximately horizontal position by means of a very thin, long band of new gray rubber, which is slipped over the rod a few *cms.* beyond *c*, towards *b* (a small groove, *g*, being cut in the upper edge to receive it), and passed, vertically below, round a hook projecting from the support or the wooden base (as in *Fig.* 56).

Behind, and close to the sharpened end *b*, is fixed a piece of curve paper pinned to a board. The position of the end *b* is marked upon it with a pencil, and distinguished by the figure 0 (zero).

Exercise XLV.

To determine the relation between the thrust of a liquid upon the base of a vessel containing it and the depth of the liquid.

Apparatus.—Pressure-balance just described, thistle-funnel with rubber base described in § 133 (*Fig.* 53), wash bottle with distilled water, box of "weights," metre scale (in *mms.*).

Method.—The funnel is fixed by a clamp upon a retort-stand so as to be exactly above the pan, and is cautiously brought down so as *just to touch* it without altering the position of the end *b* upon the curve paper, and *firmly fixed* there.

Water is now carefully introduced into the upper end of the funnel [how? see § 133], until it stands some 10 or 15 *cms.* high in the stem.

The slight strain in the rubber now makes itself manifest, the pan being slightly depressed, and the point at *b* moves a correspondingly greater distance up the curve paper. The new position of the pointer is marked off with a pencil. We have now to determine the depth of water which has produced the thrust thus recorded. To do this the metre scale is supported vertically alongside, and as near as possible to, the thistle-funnel, and the position upon it of the upper edge of the pan and the top of the water in the tube (*i.e.* the meniscus, § 118) noted as carefully as possible. In doing this the difficulty of course will be to avoid parallax (§ 13), and this may be partially met by ruling a fine horizontal line upon a piece of cardboard, and holding this behind, and close to, the scale and tube. A little practice will enable the student to judge when this line is held horizontally, and to read the positions where it coincides with the levels to be observed and the divisions of the scale with fair accuracy.

> [*Note.*—A more convenient and a safer method of avoiding parallax will be found described in § 139 below.]

The height, read with great care to the nearest millimetre, is to be carefully recorded upon the curve paper against the mark already made to mark the new position of the end *b*.

Water is again added until the column has risen another 15 *cms.* or so, and a new pair of readings of the position of *b* and the height of the column made and recorded, as above.

Finally, the tube is almost filled with water and a third pair of readings taken and recorded.

Now it remains to be found out what are the masses whose weights produce the same displacements of the rod as those produced by the various columns of water.

To do this it is only necessary to remove the thistle-funnel, notice that the point at *b* returns to the zero-mark, and then place "weights" from the box upon the pan until each of the positions successively marked is attained in turn. The three masses are then recorded alongside the heights of the water columns. It will be seen that the masses are in the same proportion as the heights.

Thus in a certain experiment the following two values were found :—

Height of Column of Water.	Mass whose Weight produced same Deflection.
(*a*) 21.6 *cms.*	18.9 *grms.*
(*b*) 31.3 ,,	27.2 ,,

The ratio of the heights is $\frac{31.3}{21.6} = 1.45$, and that of the masses $\frac{27.2}{18.9} = 1.44$.

The difference is 0.01, and the percentage difference 0.7.

Hence the height of a column, and the weight which measures its thrust, must be proportional to one another. We have for $\frac{\text{height}}{\text{mass}}$, (*a*) $\frac{21.6}{18.9} = 1.142$ and (*b*) $\frac{31.3}{27.2} = 1.150$.

The difference is 0.08, and the percentage difference as before, 0.7.

Bearing in mind the difficulty of measuring the height exactly, we may safely conclude that *the thrust of a column of water is proportional to the depth of that column.*

This result is of great importance. It can be proved, by similar means to the above, to be true for any liquid whatever.

§ 135. PRESSURE.—We have just proved that the downward thrust of a column of liquid is proportional to its depth. It can easily be shown experimentally that, so long as the base of the vessel remains the same, no alteration in the area or shape of the column of liquid has any influence upon the magnitude of this thrust, always provided the depth, measured vertically from its surface to the base, is not changed.

Thus the stem of the thistle-funnel may be bent in any direction, or narrowed down, or widened out into bulbs at various places, but—by proceeding as in the exercise just given—it can be shown that the downward thrust is still proportional to the difference in level between the upper edge of the pan and the surface of the liquid—that is to say, to the depth of the layer of liquid.

Now this thrust is felt all over the base of the vessel, and its actual value in any particular case will evidently depend upon the area of that base. It is most convenient to consider not the total thrust in any given case, but the thrust, say, in grammes-weight, per unit of area.

The *thrust per unit area* is called *pressure.*

Pressure is thus a quantity which requires for its measurement (1) a unit of force (for which we may conveniently choose here the weight of a gramme mass) and (2) a unit of area. Every pressure must be expressed as equal to so many units of force per unit of area, and the student must be careful to name the units which are used (compare § 9).

In this connection it may be well to explain the meaning of another expression often used which, in the form in which it is commonly stated, is not easily understood. This expression is "*pressure at a point.*" Since a point has no magnitude it has no area, and therefore there can be no pressure *upon* it. But the words are "pressure *at* a point," and they have a special meaning which must be carefully noticed and remembered. They mean thrust *upon unit area of which the point in question is the centre*, or, more strictly, thrust upon a unit of area every point of which is at the same depth below the surface as the point in question.

It will now be possible, if these two definitions be thoroughly understood and carefully borne in mind, to realise how the pressure of a liquid can be measured.

§ 136. MEASUREMENT OF PRESSURE.—We found in Exercise XLIV. (§ 134) that the thrust of a liquid upon the base of its containing-vessel is proportional to the depth of the liquid, and, from the definition of pressure just given (§ 135), it follows that this is true of the pressure also.

The pressure at any point (§ 135) *in a liquid is proportional to the depth of that point below the surface.*

It follows, then, that we may use the depth as a measure of the pressure, and one way of expressing a pressure consists in stating the depth of the column of a particular liquid which produces it. Thus we may speak of "a pressure of so many *cms.* of water," "a pressure of so many *mms.*," etc.

But it is convenient to be able to express these values as units of force per unit of area if need be.

Now it will easily be seen that, since in measuring pressure we always refer to the same area, 1 *sq. cm.* (or if we are using British units 1 *sq. ft.*), we may regard the thrust upon this area as due to the weight of a prism of liquid, so to speak, whose base has an area of 1 *sq. cm.* (or 1 *sq. ft.*), and whose height is the depth of the area considered below the surface of the liquid, say h *cms.* (or l *ft.*). This view will be more easily realised after a study of *Fig.* 57, where the column of liquid which comes under consideration in determining the pressure at a point A is marked by dotted lines.

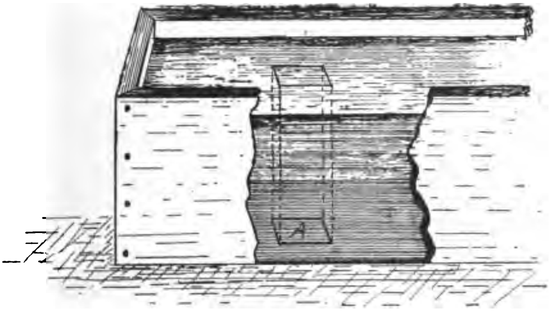

FIG. 57.

The volume of this column is evidently 1 *sq. cm.* × h *cms.* = h *c.cs.* If the density of the liquid be d *grms.* per *c.c.* the mass of h *c.cs.* is evidently hd *grms.*, and the pressure at A is the weight of hd *grms.* on each *sq. cm.*

This conclusion shows at once how, so long as we are dealing with the same liquid at the same temperature (*i.e.* so long as the value of d does not change), the pressure must depend upon the depth of the column, and thus is in agreement with our experimental results (Exercise XLIV.)

In a similar way if we desired to know the total thrust, or "whole pressure" as it is usually called, upon some area other than unity, say *s sq. cms.*, we should have to consider a column of liquid of area *s sq. cms.*, and height *h cms.* The volume of this column will be *hs c.cs.* and (the density being, as before, *d grms.* per *c.c.*) its mass *hsd grms.* The whole pressure upon the given area will then be the weight of *hsd grms.*

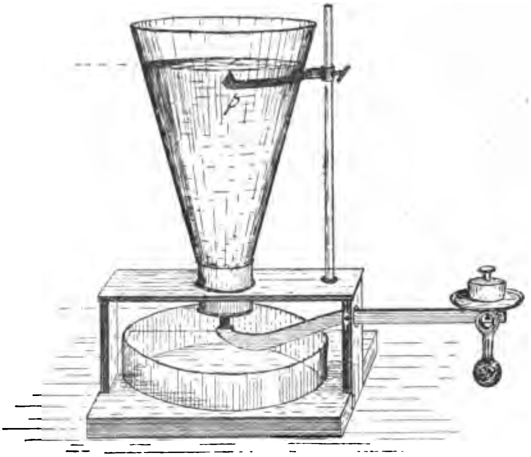

FIG. 58 (*a*).

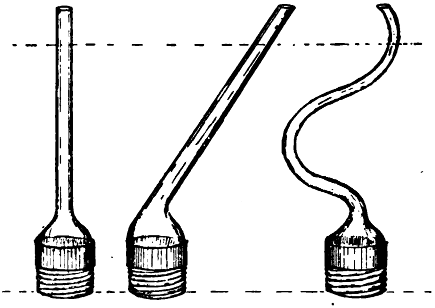

FIG. 58 (*b*).

Now this last conclusion can be, and has been, elaborately tested experimentally. Unfortunately the apparatus required is somewhat costly and ill-suited for use here, but the principle of it will be easily understood.

It consists of a series of glass vessels of various shapes, cylindrical, conical, etc., as in *Fig.* 58 (*b*), each of which has, at the lower end, an orifice of the same size, capable of being closed by a movable base which is hung from one arm of a delicate balance, and is kept pressed up against the orifice by the weight of standard masses placed in a pan attached to the other arm, *Fig.* 58 (*a*).

On very gradually filling the vessel with any liquid a thrust is established upon this movable base which, when the liquid reaches a certain height (marked by a pointer *p*), becomes just great enough to overbalance the weight acting upon the other arm of the balance. The base then begins to fall away and the contents of the vessel to run out. Now it is found with this apparatus that the shape of the vessel is quite immaterial. As soon as the depth reaches such a value that the mass represented by depth × area of base × density of liquid is never so little greater than the mass in the scale pan the base begins to fall away.

Hence we must accept the conclusion to which we have already come, namely, that *the pressure of a column of liquid whose depth is h cms. is hd grms.-weight* per *sq. cm.*, if the density of that liquid be *d grms.* per *c.c.*

PROBLEMS :—

(14) The density of benzene being 55.62 *lbs.* per *cub. ft.*, find (in pounds-weight) the whole pressure upon an area of 36 *sq. ins.*, every point of which is at a depth of 8 *ft.* below the surface of that liquid.

(15) At what depth will the pressure in water, at 4° C., amount to 1 *kilogrm.* per *sq. decimetre?*

(16) Given that the density of mercury is 13.59 *grms.* per *c.c.*, find the pressure, in *grms.-wt.* per *sq. cm.*, due to a column of mercury 760 *mms.* high.

(17) The density of chloroform being 1.5 *grms.* per *c.c.*, and that of carbon disulphide 1.3 *grms.* per *c.c.*, find at what depth the pressure in a vessel of chloroform is equal to that in a vessel of carbon disulphide at a depth of 30 *cms.*

§ 137. BALANCING COLUMNS OF LIQUIDS. — Having now assured ourselves that the pressure at any given point below the surface of a liquid depends only on the depth of that point and the density of the liquid, we may make use of this fact to compare the densities of liquids. It follows, from what we have already seen, that the heights of two columns of liquid which balance one another must be inversely proportional to their densities. This can be shown experimentally by means of an ordinary U-tube, provided we are dealing with liquids which do not mix. For example, *Fig.* 59 represents two liquids that do not mix—distinguished by different shading—contained in the two limbs of a U-tube. The place where they meet, their common surface, is marked a.

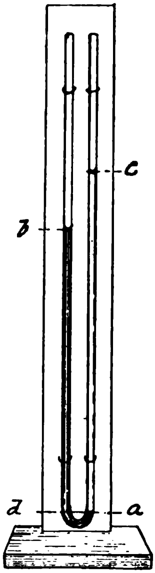

FIG. 59.

It is easily seen that the pressure at any point in this surface, a, is equal to that at any point in the same horizontal plane, d, in the other limb, for if it be not so one of two things must happen. Either the portion of liquid between a and d will move (from a towards d or d towards a —compare § 81) or this portion of liquid must be able to withstand a shear, which, as experience shows, a fluid is unable to do (compare § 91).

Hence we must admit that the pressure at a is equal to the pressure at d.

Now the pressure at a is due to the column of liquid ca. Let the height of this be h *cms.* and the density of the liquid s *grms.* per *c.c.* The pressure will then be (§ 136) hs *grms.-wt.* per *sq. cm.*

Similarly, the pressure at d is due to the column of liquid bd. Let the height of this be k *cms.* and the density of the liquid p *grms.* per *c.c.* The pressure will be kp *grms.-wt.* per *sq. cm.*

We have concluded above that these will be equal, *i.e.* that $hs = kp$, and we have now to put this conclusion to an experimental test.

Exercise XLVI.

To show that the pressure at the surface of separation of two communicating liquids is equal to the vertical depth of that surface below the surface of each liquid × the density of that liquid.

Apparatus.—U-tube with limbs about 50 *cms.* long, liquids whose density is known from experiments of Exercise XXXIX. (§ 122), say, water and olive oil, metre scale (in *mms.*), strip of cardboard, retort-stand and clamps.

Method.—A little water is first introduced into one limb so as to fill the bend, then a little olive oil into the other limb. Small quantities of water and oil are added in turn to each limb till the liquids stand at a convenient height (not less than 20 *cms.*) above the bend, and the common surface stands a few *cms.* up one limb. Fix a piece of cardboard, with a straight horizontal edge, behind and close to the U-tube, so that the upper edge runs exactly behind the surface of separation and so marks the level of this surface upon the other limb.

Then with the scale, held very close to the tube, so as to avoid parallax, measure the heights from the edge of the cardboard to the tops of the columns, reading carefully to the nearest *mm.*, and record them.

Add a little water or oil to the one or the other column, readjust the cardboard, and take measurements as before.

From the pairs of heights thus found, and the densities, calculate the pressures.

Do the results bear out the conclusion reached above ($hs = pk$)? State the conclusion clearly in words.

[*Note.*—In comparing these pressures it is necessary to bear in mind what has been said about doubtful figures in § 45. (Compare also § 109.)]

The student will now be able to solve the problem presented in the next exercise.

Exercise XLVII.

To determine the density of paraffin oil by observing the heights of balancing columns of this liquid and of water.

Apparatus.—U-tube, etc., as in last exercise.

Method.—To be devised by the student.

§ 138. HARE'S APPARATUS.—The plan adopted in § 137 is fairly convenient so long as the liquids under investigation do not mix. But there are very many pairs of liquids which, when they come in contact, do not give a surface of separation at all, but merely mix to form what is called a solution. In such a case, the method we have used above evidently cannot be adopted.

To meet the difficulty thus presented an apparatus—called from the name of its inventor Hare's Apparatus—is used, a convenient form of which is shown in *Fig.* 60.

It consists of an inverted U-tube, the open ends of which dip into two beakers *b* and *d*. To the bend is fused a tube *f*, provided with a glass tap *t*.

If now two liquids be placed in the beakers *b* and *d* it will evidently be possible to cause these liquids to rise more or less in the limbs by sucking gently at the end of a piece of indiarubber tubing slipped over the end of the tube *f*, the tap *t* being meanwhile open. The reason for the rise of the liquids is given fully in Chapter XI. Meanwhile, it will be recognised that the process is like that of filling a pipette. It will also be remembered that the contents of a pipette do not run out while the top is kept closed, and so here, if, when the liquids have risen some way in the limbs, the tap at *t* be closed, it will be found that the two columns remain stationary.

[*Note.*—It is very important that the tap *t* should fit tightly. To ensure this the barrel is removed, well dried and very thinly and uniformly smeared with vaseline. The socket is also well dried. On replacing the barrel and turning it a few times it will probably be found to fit air-tight.]

It will be proved, in Chapter XI., that the pressures upon the surface of the liquids in the beakers are sensibly equal (being that of the atmosphere—compare §§ 151 and 158). Now each of these pressures is, from the results of § 137, equal to that at a point in the same horizontal plane inside the tube.

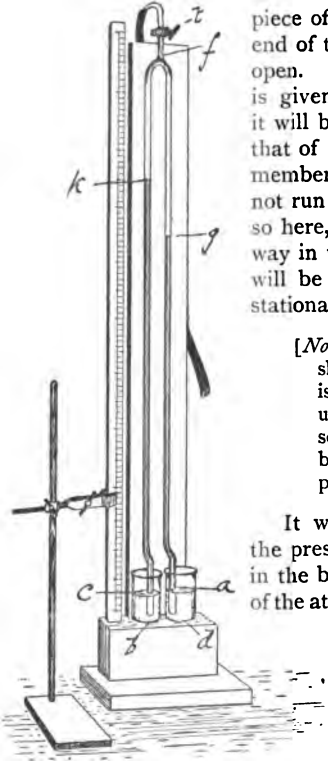

FIG. 60.

Taking the pressure at *a* (*Fig.* 60), we see that it is equal to *hs grms.-wt.* per *sq. cm.* if the height of the column *ag* be *h cms.* and the density of the liquid (in *d*) *s grms.* per *c.c.*

Similarly, the pressure at *c* is *lp grms.-wt.* per *sq. cm.* if the height of the column *ck* be *l cms.*, and the density of the liquid (in *b*) *p grms.* per *c.c.* But these are equal (as is proved below), and hence

$$hs = lp.$$

This statement can be proved experimentally by taking two liquids whose densities are already known and measuring the heights of the columns (see Exercise XLVII. § 139), but before this can be done successfully it is necessary to indicate a better method than we have used hitherto for the measurement of vertical heights.

§ 139. MEASUREMENT OF VERTICAL HEIGHTS—READING-TELESCOPE.—The apparatus most conveniently used for the measurement of vertical heights consists of a small telescope mounted so that it can be moved up and down as a whole, being then fixed in any desired position by a screw or screws, and can be rotated in a horizontal plane. The telescope consists essentially of two "lenses" of glass, a smaller one known as the eye-piece, and a larger known as the object-glass. By altering the distance between the eye-piece and the object-glass it becomes possible to see clearly things at different distances from the object-glass. To facilitate this movement the two lenses are mounted in the ends of two brass tubes, of which the smaller, which carries the eye-piece, slides to and fro inside the larger, its movements being usually, and most conveniently, controlled by a screw with a milled head. Inside the smaller tube, and at such a distance from the eye-piece that on looking through it they are seen perfectly sharp and clear, are two fine cross-wires, the point of intersection of which gives a convenient fixed point of reference. The tube which carries the eye-piece is made double, so that the distance of this lens from the cross-wires can be adjusted to suit the eye of each observer before the larger movement (which shifts both eye-piece and cross-wires together), necessary to obtain a clear view of the object towards which the telescope is directed, is begun. The adjustment of these distances is called "focussing," and must be carefully done before an attempt is made to use the telescope. [For further explanations of the construction and mode of action of the telescope, the student must wait until he takes up the study of Optics.]

A simple method of adapting the telescope to the measurement of vertical heights is illustrated in *Fig.* 61. The telescope is supported, with its axis horizontal, in a wooden carrier, which can slide up and down a vertical pillar *p*. The movements of this carrier are controlled by the screws *s*, *s*, which are adjusted so that the carrier just slides stiffly. At one side of this carrier is another screw (not visible in the figure), which serves firmly to fix it in any desired position. The central pillar springs from a circular iron base, *b*, which rests upon three thin pieces of cork or sheet-rubber (of equal thickness) placed on the top of a circular piece of plate glass *c*. This piece of glass in turn rests upon a second similar disc, which is cemented on to the bottom of a circular hole in a heavy wooden block *w*. This wooden block is supported upon three levelling screws, by adjusting which the central pillar can be made to stand vertically. Between the two discs of plate glass a small quantity of lycopodium powder is introduced, so that the upper can be rotated very easily in a horizontal plane.

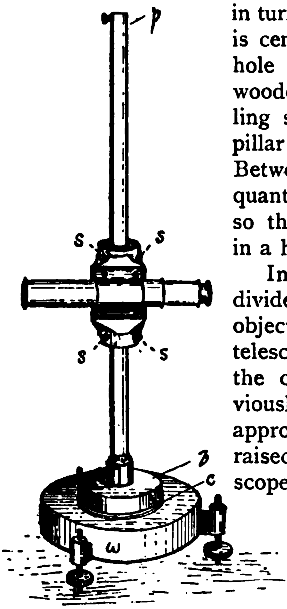

FIG. 61.

In measuring a vertical height a scale, divided in *mms.*, is fixed vertically near the object to be observed. Then the reading-telescope is set up some metres away from the object, the eye-piece having been previously focussed upon the cross-wires, and approximately levelled. The carrier is then raised or lowered until, on focussing the telescope, the point of intersection of the cross-wires is seen to coincide (as nearly as possible) with the mark upon that object whose position is to be observed. The wooden base is placed in such a position that, when the telescope is thus directed, one of the three levelling screws is vertically below the eye-piece. Then, by turning this levelling screw very slightly up or down, the point of intersection of the cross-wires is made to coincide exactly with the mark in question.

If now the telescope be simply rotated, by placing the hand on the iron base, *b*, until a portion of the scale comes into the field of vision, it is easy to read off the position of this point upon the scale, estimating (as may easily be done since the image seen through the telescope is magnified) to the tenth of a millimetre. The telescope is then once more levelled approximately, the fixing screw is loosened and the carrier raised or lowered until the second mark whose position is to be observed nearly coincides with the point of intersection of the cross-wires. The back levelling-screw is again used as a fine adjustment, and, by turning the upper glass plate upon the lower, as before, the point upon the scale, which is exactly opposite this mark, is brought into view.

From the two scale-readings thus obtained the distance between the two marks is found by subtraction.

[*Note.*—It will be seen that the attempt is here made to avoid parallax by bringing, at each observation, three points into a straight line, namely, the mark to be observed, the point of intersection of the cross-wires, and the portion of the observer's eye upon which the image of both falls. An error may arise if the distances of the object and the scale from the telescope are appreciably different (as the student will realise when he comes to the study of Optics), but if these distances be very nearly the same the error due to parallax is always less than anything we are able to detect by line measurement.]

The construction and use of the reading-telescope being now understood, it will be easy for the student to verify the statement at the end of § 138.

Exercise XLVIII.

To establish the relation between the densities of two liquids and the heights of the columns of them which balance in Hare's Apparatus.

Apparatus.—Hare's Apparatus (*Fig.* 60, § 138), reading-telescope (*Fig.* 61), metre-scale divided in *mms.*, beakers, wood blocks, retort-stand and clamp, liquids whose densities are known, *e.g.* water and alcohol.

Method.—The tap of the Hare's Apparatus is cleaned and lubricated as directed above (§ 138, note). The beakers are placed in position and nearly filled, the one with water, the other with alcohol. The liquids are caused to rise to a convenient height in the two tubes. The metre-scale is fixed vertically between and close to the tubes. The level of the liquid in each beaker and tube is then observed with the reading-telescope as described above.

Several pairs of readings should be made with columns of different heights.

Calculate, for each pair, the values of height of column × density of liquid.

Do your results agree with the statement that the heights of the columns are inversely proportional to the densities of the liquids?

The student should now make use of this means of finding the density of a liquid.

Exercise XLIX.

To find the density of brine.

Apparatus.—As for Exercise XLVIII. Brine (made by shaking about 100 *grms.* of powdered common salt, for some time, with about 250 *c.cs.* of water and filtering the solution).

Method.—To be devised by the student.

§ 140. UPTHRUST ON AN IMMERSED SOLID.—It is a familiar observation that some objects when placed in a liquid, such as water, sink to the bottom of it, while others float on its surface.

Now a little thought will show that in the latter case the floating body must be acted upon by a force directed upwards which counterbalances the downward force called the weight of that body (see §§ 81 and 90). That this is so we shall presently satisfy ourselves by direct experiment (Exercise LI. § 141). But what are we to conclude about the case of objects which sink in water? Are we to suppose that they do not experience any upward thrust? Or is it that in this case an upward thrust exists, *but is*, for some reason which we are to discover, *less than* that downward force which we call the weight of the object? We can easily decide between these alternatives by experiment.

Exercise L.

To determine whether a solid which sinks in water or other liquid experiences any upward thrust on immersion.

Apparatus.—Spring balance (§§ 93 and 97), small heavy objects, *e.g.* leaden bullet, iron washer, etc., fine thread, beaker, water, alcohol, etc.

Method.—The spring balance is adjusted and the zero marked as in Exercise XXX. (§ 97). The object is attached to a small piece of fine thread, which is tied into a loop at the free end, and is hung by this from the hook below the pan of the balance. The shear produced by its weight draws the pointer of the balance down to a certain position which is marked. Then the beaker, nearly filled with the liquid chosen, is raised from below the object until the latter hangs, completely immersed, well in the centre of the beaker.

Does the shear in the spring alter at all? In what direction? What conclusion can you draw?

What is now your answer to the question suggested at the beginning of this exercise?

§ 141. FLOATING BODIES.—Having now satisfied ourselves concerning the existence of an upthrust upon an immersed body we may proceed to test the truth of the idea which led us to seek for information on that subject.

Exercise LI.

To show that the upthrust upon an object floating in a liquid is equal to the weight of that object.

Apparatus.—Spring balance, fine thread, large beaker (or dish), water (or other liquid), small block of wood (about 5 *cms.* by 3 *cms.* by 2 *cms.* will be a convenient size. The wood must be varnished so that it may not become sodden).

Method.—The balance is adjusted and the zero marked (this has already been done in Exercise L., and it is only necessary here to verify the zero-mark). The beaker (or dish) is placed so that its centre is below the hook under the pan, and the wooden block is suspended by a fine thread so as to hang within it and just below its edge. The new position of the pointer is carefully marked. Then water (or other liquid) is very gently and slowly poured into the beaker.

What happens to the pointer when the water begins to rise round the wood? What position does the pointer finally reach?

State very carefully all the conclusions you can draw.

Does this experiment bear out the statement contained in the heading of the exercise?

We have now seen that a solid object placed in a liquid experiences an upthrust. In some cases this upthrust becomes equal to the weight of the object before the latter is entirely submerged, and the object floats. In other cases the upthrust remains less than the weight even when the object is entirely immersed, and the object sinks.

Now we have already seen (§ 130, Exercise XLI.) that an object immersed in a liquid displaces its own volume of that liquid, and it is easy to show, by an experiment similar to that of Exercise XLI., that a floating body displaces a volume of liquid which is less than its own volume.

The question suggests itself whether there may not be some connection between the weight of this "displaced" volume of liquid and the upthrust experienced by the object.

We can answer this question very easily in the case of an object which floats. In this case, as we have already proved with the spring balance (Exercise LI. § 141), the upthrust is equal to the weight of the object. If then we determine the mass of, say, the block of wood used in Exercise LI., we shall have a quantity which, at any given place, may be used as a measure of the weight, and hence also of the upthrust. The mass of the liquid displaced on floating this block may be found by means of the apparatus shown in *Fig.* 53 (§ 130).

Exercise LII.

To determine the relation between the weight of a floating body and that of the liquid displaced by it.

Apparatus.—Balance, box of weights, small block of wood (varnished), fine thread, vessel with overflow tube (as *Fig.* 53), small beakers.

Method.—Find the mass of the wooden block (§ 107). Collect the liquid displaced on floating it (with the precautions mentioned in § 130) in a tared beaker and find its mass.

Compare your results with the following statement:—*An object floats upon a liquid when the weight of liquid displaced is equal to the weight of that object.*

§ 142. MEASUREMENT OF UPTHRUST—DEDUCTION OF VOLUME.—This important fact having been established, it is necessary to find some means of measuring directly the upthrust upon an object which sinks in a given liquid and to compare this with the weight of the volume of liquid which we know (either from linear measurements or by the method of Exercise XLI.) the object has displaced.

It is evident that the simplest plan will be to determine the mass of the object, and to compare this with its apparent mass when immersed in the liquid. The difference will be a measure of the mass whose weight is equal to the upthrust.

The usual method of doing this is to suspend the object by a piece of fine thread of suitable length from the hook on the under side of the stirrup which carries the pan of the balance and determine its mass by direct weighing (§ 101). A small wooden bridge, wide enough to stand clear of the pan and high enough to allow the pan a fair swing when the beam is raised clear of the arrestment, is placed across the pan below the object, and upon this rests a beaker, small enough to stand easily inside the frame which carries the pan and deep enough to allow the object to hang within it, 3 or 4 *cms.* below the edge, even when the beam swings into its extreme position. If this beaker be almost filled with some liquid in which the object will sink, it is now easy to determine (again by direct weighing, § 101) the mass needed to balance the object under these new conditions. The difference between these two masses evidently gives the mass whose weight is equal to the upthrust.

The arrangement of the bridge, etc., is shown in *Fig.* 62.

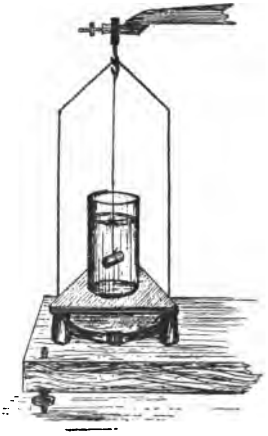

FIG. 62.

Exercise LIII.

To measure the upthrust upon a solid of known volume wholly immersed in a liquid of known density.

Apparatus.—Objects whose volume has been determined as in Exercise XXIV. (§ 69) or Exercise XLII. (§ 130), liquids whose densities are known (from results of Exercise XXXIX. § 112), balance, box of "weights," bridge, beaker, fine thread, small camel's-hair paint-brush with long handle.

Method.—The object is hung as described from one arm of the balance and its mass is determined (§ 101). Then the beaker is filled, to within about 2 *cms.* of the top, with the liquid, and the apparent mass of the object as it hangs in the liquid is determined as before. Care must be taken that no bubbles of air adhere to the object, and any which form must be removed by touching them with the camel's-hair brush. Take the difference between these two masses and record it as the upthrust in grammes-weight.

The volume of the object and the density of the liquid being known, calculate the mass of the liquid displaced. How does it compare with the mass whose weight equals the upthrust?

[*Note.*—In the above exercise no account has been taken of the mass of the thread used to suspend the object, nor of the volume of that part of it which, in the second weighing, is immersed in the liquid. If the thread be fine, and as little as possible of it be used, the error from these causes is very slight, and may be neglected here. In accurate work a hair should be used for suspension, but in that case much practice is needed successfully to suspend the object. We have also used the least accurate method of weighing. In some cases a special pan is supplied with the balance, having the same mass as the ordinary pan, but a much shorter frame, so that the object can be suspended from a hook beneath it. Then the more accurate method of weighing (§ 107) can be used.]

From the results obtained with various objects and different liquids test the following statement:—

The upthrust upon an object wholly immersed in a liquid is equal to the weight of the volume of the liquid displaced by that object.

It is plain that if this conclusion be once surely established we have gained a means, far more accurate than that used in Exercise XLII. (§ 130), of measuring the volume of a solid of any form.

This method consists in three steps: (1) determining the mass of the solid (in air—see also § 161), (2) determining its apparent mass when wholly immersed in some liquid whose density is known and which does not dissolve or otherwise act upon it, and (3) calculating from the difference between these two masses, which is clearly the mass of the liquid displaced, the volume of that liquid, and hence of the solid which has displaced it.

To ensure that this is well understood the student should, before going on to use the method, work out the following problems:—

(18) An object has a mass of 37.4 *grms.* When wholly immersed in chloroform (density 1.5 *grms.* per *c.c.*) its apparent mass is 32.3 *grms.* What is the volume of the object? What would be its apparent mass in water at 4° C.?

(19) A certain object when weighed in water showed a loss of 15 *grms.* weight. When weighed in another liquid it showed a loss of 12 *grms.* weight. What was the volume of the object and what the density of the other liquid?

(20) Two objects *A* and *B* were taken. The mass of *A* was 25 *grms.*, and its apparent mass in water at 4° C. was 10 *grms.* The mass of *B* was 20 *grms.*, and its apparent mass in benzene (density 0.9 *grm.* per *c.c.*) was 6.5 *grms.* Compare the volumes of *A* and *B*.

We may now proceed to use this method to find the volume of any given object. It is evident that, since the density of water is not at ordinary temperatures very far removed from 1 *grm.* per *c.c.*, it will be most convenient, especially as we do not require extreme accuracy, to use *water* whenever practicable, for the calculation of the volume from the observed upthrust is thereby much simplified.

For ordinary purposes then we may take the density of water as 1 *grm.* per *c.c.* and consider the number of *grms.-wt.* in the upthrust as equal to the number of *c.cs.* in the volume of the object.

Exercise LIV.

To measure indirectly the volume of an object by observing the upthrust upon it when immersed in a liquid (such as water) in which it sinks.

Apparatus.—Balance, box of "weights," bridge, beaker, fine thread, small camel's-hair brush, objects to be measured (*e.g.* the one used in Exercise XLII. § 130, penny, etc.), water.

Method.—As in Exercise LIII. [Caution. When an object of irregular shape is to be used it is particularly necessary to beware of adhering air-bubbles.]

In case a solid is acted upon by water it will evidently be necessary to choose some other liquid in which it will sink, determine the density of that liquid (with the specific gravity bottle or pyknometer), and then determine the upthrust and calculate the volume as above.

The student should take, for example, a piece of rock-salt (which would be dissolved by water) and find its volume by weighing in air, and, say, benzene.

§ 143. INDIRECT MEASUREMENTS OF LINEAR EXTENSION.—It is evident that the possibility of thus finding the volume of an object, whatever its shape, will enable us to make various other measurements indirectly. We have seen, in §§ 68 and 71, how the volumes of solids of various regular forms are connected with certain linear dimensions of them, and it has been indicated briefly, in § 111, how, given a knowledge of the volume and of some of these linear dimensions, it is possible to calculate the rest.

The application of these indirect methods will now be possible on an extended scale.

Thus, for example, a piece of uniform *wire* is, as already pointed out, a cylinder. If the length of a piece of wire be l *cms.*, and its radius r *cms.*, its volume is $\pi r^2 l$ *c.cs.* (§ 68). Now supposing the wire to be very thin we may have some difficulty in measuring the diameter (in this case $2r$ *cms.*) directly, but it is easy, since we may make the length what we please to measure that with considerable accuracy, and the volume of the piece of wire ($\pi r^2 l$ *c.cs.*) can be found by rolling it up and weighing it first in air and then in water.

From these two the value of r (in *cms.*), and hence the diameter, can be calculated.

Again, suppose we have to deal with a thin sheet of metal, say a piece of tinfoil. We can easily cut out a rectangular piece whose length (l *cms.*) and breadth (b *cms.*) can be measured with the necessary exactness. This piece is a rectangular prism whose volume, if the thickness be h *cms.*, is lbh *c.cs.* (§ 68). Now by folding up the sheet of tinfoil to a suitable size it is not difficult to find this volume by weighing in air and then in water, and from this, and the known length and breadth, to calculate the thickness.

Exercise LV.

To determine the diameter of a fine wire.

Apparatus.—Metre scale graduated in *mms.*, balance, box of "weights," bridge, beaker, fine thread, small camel's-hair brush, fine wire, wire-cutters.

Method.—Straighten a portion of the wire; measure and cut off accurately a definite length of it, say 50 *cms.* Roll this up, suspend it by a fine thread above one pan of the balance, and find its mass (§ 101). Then allow it to hang in water, taking care to remove all air-bubbles, and again determine the mass. From the results deduce the volume and thence calculate the diameter.

The result may be checked by direct measurement with the micrometer (as in Exercise XIV. § 34).

To ensure that he understands the principle and practice of the method just used, the student should now solve the problems presented in—

Exercise LVI.

To determine the volume and the average thickness of a coin, such as a penny or a threepenny piece.

Apparatus.—As for last exercise, but, instead of wire and cutters, coin and callipers.

Method.—To be devised by the student.

§ 144. VOLUME AND SURFACE AREA OF SPHERE.—It was pointed out, in § 74, that the volume of a sphere must be regarded as equal to that of a very great number of pyramids whose height is the radius of the sphere, and the areas of whose bases are together equal to the surface of the sphere. Now the area of the surface was stated in § 59 to be $4\pi r^2$ *sq. cms.* where *r. cms.* is the radius. This gave us for the volume $\frac{1}{3}$ of $4\pi r^2$ *sq. cms.* × *r cms.*, or $\frac{4}{3}\pi r^3$ *c.cs.* (compare § 71).

As we now have a means of finding the volume with a very fair degree of accuracy, we can test this conclusion and satisfy ourselves as to the truth, or otherwise, of the statement in § 59.

Exercise LVII.

To verify the relation between the diameter and volume of a sphere.

Apparatus.—Metal sphere, callipers, *mm.* scale, balance, "weights," bridge, beaker, fine thread, camel's-hair brush.

Method.—Determine the diameter as accurately as possible. Find the volume from the upthrust. Calculate the volume from the diameter, and compare with that found from the upthrust.

§ 145. SPECIFIC GRAVITY OF AN OBJECT WHICH WILL SINK IN WATER.—Specific gravity was defined, in § 123, as the ratio of the density of a substance to the density of some other substance chosen as a standard, and it was mentioned that the standard substance always chosen is *water*. This being so it will be clear that the specific gravity of any object which will sink in water can be found at once by determining the mass of that object first in air and then when suspended in water. The difference is the mass of water displaced by the object—that is to say, is the mass of a volume of water equal to the volume of the object (§§ 130 and 142). Taking the density of water as known, this gives us the volume of the object, and we have then for the specific gravity of the object—

$$\frac{\text{Density of object}}{\text{Density of water}}=\frac{\dfrac{\text{mass of object}}{\text{volume of object}}}{\dfrac{\text{mass of water displaced}}{\text{volume of water displaced}}}=\frac{\text{mass of object}}{\text{mass of water displaced}}.$$

The calculation may then be simplified, and it may be stated as a general result that the specific gravity of an object can be found by dividing its mass (as found with the balance) by the apparent loss of mass it experiences when immersed in water.

This gives an important and most convenient way of measuring specific gravity, but it must not be supposed to afford a *definition* of specific gravity.

It is evidently a matter of complete indifference what unit of mass we may choose. [Why? Compare § 123.]

PROBLEMS :—

(21) A cannon ball has a mass of 45 *lb.* When suspended in water its apparent mass is 39 *lb.* What is its specific gravity? Taking the density of water as 62.5 *lb.* per *cub. ft.*, find the density of the cannon ball.

(22) The specific gravity of pure silver may be taken as 10.5. A certain solid object has a mass of 32 *grms.*, and its apparent mass in water is 28 *grms.* Does it consist of pure silver?

(23) A 50-*grm.* brass "weight," of which the specific gravity is 8, is suspended in water from one end of the balance beam. At the other end of the beam a piece of platinum is suspended, also in water, and is found just to balance the brass "weight." The mass of the platinum is 45.9 *grms.* What is the specific gravity of the platinum?

[*Note.*—It was mentioned, in § 112 above, that Archimedes of Syracuse was the first to determine the relative density of a substance in order to test its purity. It was he who devised the plan of measuring specific gravity which has just been described, and the general rule that an object on immersion in a liquid suffers a loss of weight equal to the weight of the liquid displaced is known as the *Principle of Archimedes.* According to account, Archimedes was asked to find out whether one of Hiero's crowns was of pure gold, or of gold alloyed with silver. He is said to have hit upon the solution of the problem through noticing the upthrust upon his own body when it was immersed in his bath. Thereupon he compared the loss of weight experienced by equal masses of pure silver and pure gold on immersion in water. He found that the silver weighed less in water than the gold, and then, having shown that the king's crown weighed less in water than an equal mass of pure gold, he was able to affirm that it contained some other metal.]

§ 146. USE OF SINKER.—It is manifest that if the mass and volume of an object be determined, and that object be then weighed in a liquid of unknown density, it is possible directly to calculate, from the observed upthrust, the *density* of that liquid.

Any object may be so used which will sink in the liquid to be examined, and is not dissolved or attacked by it, and an object so employed is often called a *sinker.* A convenient form is shown in *Fig.* 63. It is constructed from a thick piece of glass rod, which is rounded at one end and drawn out at the other, the drawn-out portion being bent into a ring or hook.

The volume of the sinker is found most easily of course by weighing in air and then in water (§ 142). Then it is weighed in the liquid whose density is to be found. The apparent loss of mass gives the mass of a known volume of the liquid, and hence its density is calculated.

PROBLEM :—

(24) A glass sinker had a mass of 45 *grms.* When suspended in water it appeared to weigh 30 *grms.*, and in oil of turpentine 32.25 *grms.* What was the density of the oil of turpentine in *kilogrms.* per *litre?*

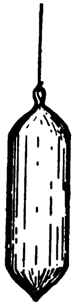

FIG. 63.

It is also very easy to see how from the three weighings with a sinker the *specific gravity* of a liquid can be found. The "loss of weight" in water gives the mass of water displaced. The "loss of weight" in the liquid gives the mass of the same volume of that liquid. The ratio of the latter to the former evidently gives the specific gravity of the liquid.

PROBLEM :—

(25) A sinker weighed 21 *ozs.* in air, 18 *ozs.* in water, and 17.2 *ozs.* in glycerine. What is the specific gravity of glycerine?

But a sinker may be used for yet another purpose.

The method which we have developed for finding specific gravity (and density) is not, in the form we have stated it, perfectly general, for it is applicable only to objects which will sink in water (§ 145).

It is, however, quite easy, by the help of a suitable sinker, to apply it to objects which naturally float upon water. A little thought will show that if we take a sinker heavy enough and attach it to such an object we can cause that object to sink, and then, by measuring the upthrust upon sinker and object together we can, knowing already from a separate experiment the upthrust upon the sinker itself, easily deduce the upthrust upon the object.

Thus, for example, suppose we have a sinker whose mass is *m grms.*, and let its apparent mass in water be *a grms.*, then evidently the upthrust upon this sinker is $(m - a)$ *grms.-wt.*

If now we take an object whose mass is *w grms.* and attach the sinker to it we shall have a total mass, for sinker and object, of $(w + m)$ *grms.* Finally, suppose the apparent mass of sinker and object together in water is b *grms.* The upthrust upon the two together is manifestly $(w+m)-b$ *grms.-wt.* The difference between this and the upthrust upon the sinker alone, namely, $[(w+m)-b]-[m-a]$ is clearly the upthrust in *grms.-wt.* upon the object itself. Calling this quantity p *grms.-wt.* we have: mass of object $= w$ *grms.;* mass of equal volume of water $= p$ *grms.;* and hence specific gravity of object $=\frac{w}{p}$.

PROBLEM :—

(26) A piece of cork whose mass is 12 *grms.* is attached to a leaden sinker whose mass is 66 *grms.* The two together, when immersed in water, have an apparent mass of 20 *grms.* The sinker alone has an apparent mass, in water, of 56 *grms.* What is the specific gravity of the cork?

Exercise LVIII.

To find the specific gravity of a solid which floats upon water.

Apparatus.—Balance, "weights," bridge, beaker, fine thread, camel's-hair brush, piece of sheet-lead to serve as sinker, solid whose specific gravity is to be found, *e.g.* small piece of wood or cork or paraffin wax.

Method.—First determine the mass of the sinker and then its apparent mass in water (as in Exercises LIII. and LIV. § 142). Then dry the sinker and attach it to the object under investigation; determine the mass of the two together (and hence deduce the mass of the object, and also their apparent mass in water.

Calculate the specific gravity as above.

§ 147. SUMMARY OF METHODS FOR DETERMINING DENSITY AND SPECIFIC GRAVITY.—The student should now sum up carefully for himself all the methods which we have shown to be available for determining the densities and specific gravities of objects.

The following hints and reminders will help him to do this successfully:—

The determination of *density* always involves *two distinct measurements:* (1) Measurement of *mass.* This is always carried out, in practice, with the common balance, and the various methods of doing it are described in §§ 101, 103, 104, and 107. Of these methods that of § 107 is always to be used when possible. (2) Measurement of *volume.* This may be effected in a variety of ways: (*a*) in case of solids, by linear measurements, §§ 68 and

71; (*b*) in case of liquids, by use of graduated vessels, § 122; (*c*) in case of solids, by observations, direct or indirect, of volume of water or other liquid displaced by the solid, § 130; (*d*) in case of solids, by observation of the upthrust upon the solid when completely immersed in a liquid of known density, § 142, with use, if necessary, of a sinker, § 146.

It is also possible to compare the densities of two liquids, and hence, if the density of one be known to calculate that of the other, either by the use of the pyknometer, § 126, or by observations of the heights of balancing columns, §§ 137-139.

The determination of *specific gravity* involves the comparison of two densities, that of the substance under consideration, and that of some, chosen, standard substance. The standard substance chosen is water. All the methods available for finding densities are therefore also available for finding specific gravity. [Illustrate this.]

Further, if in the methods of § 126 (pyknometer) or of §§ 137-139 (balancing columns), one of the liquids employed be water, the results give the specific gravity of the other liquid easily and directly.

Lastly, the general method of § 145, with, if necessary, the slight modification mentioned in § 146, will enable the specific gravity of any solid to be found directly, while the use of a suitable sinker—§ 146—enables a similar simplification to be made in the case of liquids.

The student should make sure that he understands thoroughly the fact that the density (in any chosen units) of any substance can be found by multiplying the density of water (in the same units) by the specific gravity of that substance.

[*Note.*—The influence of temperature upon density is, in this summary, ignored, for the sake of simplicity.]

§ 148. NICHOLSON'S HYDROMETER.—This instrument is simply a hollow object of suitable shape which floats in ordinary liquids, and which can be weighted by the addition of standard masses, so as in all cases to be immersed exactly up to a definite mark. It is thus a floating body whose mass can be varied at will. When submerged up to the mark it always displaces the same volume of any liquid. This volume can be found once for all by floating it, weighted as may be necessary, in water. Then by observing the mass necessary to be added to that of the instrument to immerse it to the same extent in another liquid, the specific gravity of that other liquid can be deduced. The common form of it is shown in *Fig.* 64. The body of the instrument is a hollow metal cylinder, to the lower extremity of which is rigidly attached a heavy weight, *p*, in shape an inverted cone, the upturned base of which is slightly concave. From the upper end of the body rises a thin metal stem, which has engraved upon it a mark, *m*, and which carries above it a small scale-pan.

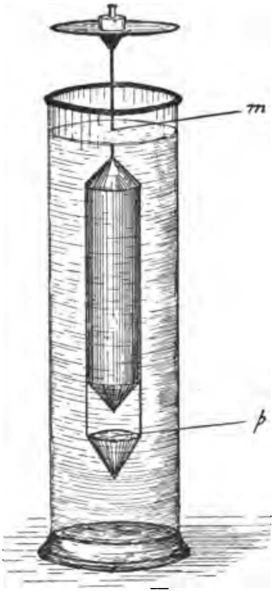

FIG. 64.

The conical weight fixed below the body of the instrument serves a double purpose. In the first place, it causes the cylinder to float upright, instead of on its side; and, in the second place, the concave base serves as an additional scale-pan, the use of which will be explained below.

It will be easily understood how this instrument can be used to find the specific gravity of a liquid. Suppose that the mass of the hydrometer is found to be *W grms.*, and that when floating in water it requires the addition upon the upper scale-pan of *m. grms.* to bring the mark upon the stem into the surface. Then the mass of a volume of water equal to that of the hydrometer up to this mark is (§ 141) $[W+m]$ *grms.* Now let the instrument be dried and placed in some other liquid, and suppose the addition of *p grms.* on the upper pan is now necessary to sink it as before.

Then the mass of a volume of that liquid equal to that of the hydrometer up to the mark is (§ 141) $[W+p]$ *grms.*

Hence we have

$$\text{Specific gravity of liquid} = \frac{\text{mass of certain volume of liquid}}{\text{mass of equal volume of water}} = \frac{W+p}{W+m}$$

(compare § 145).

PROBLEM :—

(27) A Nicholson's hydrometer has a mass of 67 *grms.* To sink it to the mark in water an addition to the upper pan of 23 *grms.* is required, and, to sink it in naphtha, 8.6 *grms.* What is the specific gravity of the naphtha ?

Exercise LIX.

To find the specific gravity of a liquid by means of a Nicholson's hydrometer.

Apparatus.—Nicholson's hydrometer, tall glass jars, box of "weights," small camel's-hair brush, water, cardboard cut as in *Fig.* 65, methylated spirits or other liquid, balance, counterpoise.

Method.—(1) Determine the mass of the hydrometer. (2) Float it in the cylinder of water and find out what weights are required to bring the mark upon the stem into the surface of the water, taking care to avoid parallax in reading. It is necessary to be very careful not to add so large a mass that the whole hydrometer is immersed, and to avoid this danger the student should place over the top of the glass cylinder a piece of cardboard cut as shown in *Fig.* 65. If the cylinder be so filled that when the hydrometer is placed in it the liquid is within a few *cms.* of the top, and care be taken that the stem nowhere touches this piece of cardboard, it is easy to get accurate readings; and at the same time, even should the pan be overloaded, there is no danger of its being submerged. It is also important that, as the hydrometer is introduced into the liquid, all air-bubbles should be carefully removed. (3) Remove the instrument from the water, dry it, and again, with the precautions named above, find the mass required to sink it in the other liquid. Calculate the specific gravity of the other liquid as above. Notice that one determination of the mass required in water will serve for a whole series of determinations with other liquids.

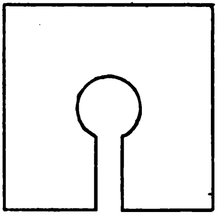

FIG. 65.

[*Note.*—The name *hydrometer* is derived from the Greek words for "water" and "measure," and thus comes to be applied to an instrument for comparing other liquids with water.]

Not only, however, can Nicholson's Hydrometer be used to measure the specific gravities of liquids, but it also affords a very easy way of measuring the densities and specific gravities of solids. For suppose we know the mass required to sink the hydrometer to the mark in water, say *m. grms.* Let this be removed, and let the object (which must evidently not have a greater mass than *m. grms.*) be placed in the upper pan. Suppose the mass now needed to sink the hydrometer to be p *grms.* Then it is evident that the mass of the object is $(m - p)$ *grms.* If the object be now transferred to the lower pan and the mass needed upon the upper be again determined, say *w. grms.*, it is evident that the apparent mass of the object when immersed in water is $(m - w.)$ *grms.*

Hence the "loss of weight," = mass of object, *minus* apparent mass in water, $= (m - p.) - (m - w.)$ *grms.* $= (w - p.)$ *grms.*

From this the specific gravity of the object—

$$= \frac{\text{mass of object}}{\text{mass of equal volume of water}} = \frac{(m - p)}{(w - p)}$$

[It will be evident that if the object be one which floats upon water it will need to be tied down to the lower pan by means of a fine thread, the mass of which is small enough to be neglected.]

Before using this instrument the student will do well to make sure that he understands thoroughly the principle involved, and he should test his understanding of it by solving the following problems :—

(28) The mass needed to sink a Nicholson's hydrometer to the mark in water was 5.1 *ozs.* An object was placed in the upper pan, and the mass then needed was 1.5 *ozs.*, while with the object in the lower pan 2.4 *ozs.* were required. What was the specific gravity of the object? Given that the density of water is 1000 *ozs.* per *cub. ft.*, calculate the volume of the object (in *cub. ft.*) and its density (in *ozs.* per *cub. ft.*).

(29) The density of a certain object is 0.5 *grms.* per *c.c.* When it was placed in the upper pan of a Nicholson's hydrometer the addition of 15 *grms.* is needed to sink the instrument to the mark. Without the object 20 *grms.* are needed. Find what mass must be placed in the upper pan when that object is attached to the lower pan.

(30) Explain carefully how it is possible to find the specific gravity of a liquid, such as alcohol, with a Nicholson's hydrometer, without knowing the mass of the instrument itself by the use of, say, a leaden bullet as a sinker (§ 146).

(31) When a certain object is placed in the upper pan of a Nicholson's hydrometer 4.1 *grms.* are needed to sink it in water and 34.1 *grms.* to sink it in carbon disulphide. With the object in the lower pan, 11.1 *grms.* and 43.2 *grms.* are needed in water and carbon disulphide respectively.

Calculate the specific gravity of carbon disulphide.

Exercise LX.

To find the specific gravity of a solid by means of Nicholson's hydrometer.

Apparatus.—Nicholson's hydrometer, glass jar, cardboard cover (*Fig.* 65) water, box of "weights," objects whose specific gravity is to be found, camel's-hair brush.

Method.—(1) Determine the mass needed to sink the hydrometer to the mark; (2) place the object in the upper pan and again determine the mass needed; (3) place the object in the lower pan, fastening it there if it would float on water with a small piece of fine thread, and once more find the mass needed to sink the instrument.

Calculate the specific gravity of the object as above.

Exercise LXI.

To find the specific gravity of a liquid by means of Nicholson's hydrometer without determining the mass of the instrument.

Apparatus.—As for Exercise LX., but, instead of objects whose specific gravity is sought, a sinker of glass or metal, liquid whose specific gravity is to be found.

Method.—(1) Place the hydrometer in water and find the masses needed to sink it with the sinker (*a*) in the upper pan, (*b*) in the lower pan. The difference is the mass of water displaced by the object. (2) Carefully dry the hydrometer and sinker. (3) Place in the second liquid and find the mass required to sink the instrument (*c*) with the sinker in the upper pan, (*d*) with it in the lower pan. The difference is the mass of this liquid displaced by the object.

From the data thus obtained calculate the specific gravity of the liquid.

[*Note.*—The Nicholson's hydrometer being constructed of metal, usually of brass, cannot be used with a very large number of liquids, since it would be corroded by them. A hydrometer is made which is exactly the same in principle, but is constructed of glass, and this of course can be much more widely used. This form is known as Fahrenheit's hydrometer.]

§ 149. HYDROMETERS OF VARIABLE IMMERSION.—As already pointed out the principle of Nicholson's hydrometer is that, by means of a variable load, the instrument is always immersed to the same extent. It is manifestly possible, and in some cases more convenient, to keep the mass of the hydrometer itself constant, and to measure the extent to which it sinks in different liquids. A number of hydrometers embodying this principle are in common use, and are known by various names, sometimes those of their inventors, sometimes expressive of the particular purpose for which they are to be used, as, for example, "alcoholimeters," used for comparing the strengths of alcoholic liquors by observations of their specific gravities, and so on.

They all consist essentially of a thin glass tube, having, usually, two bulbs close together at one end. The lower of these bulbs is small and contains some mercury, the upper is larger and contains only air. The other end of the tube is sealed. When such an instrument is placed in water, or other suitable liquid, it floats upright with the weighted bulb at the bottom, and by adjusting the quantity of mercury in this it can be so arranged that the narrow stem is submerged to any desired extent. One form, known as Twaddell's hydrometer, is shown in *Fig.* 66.

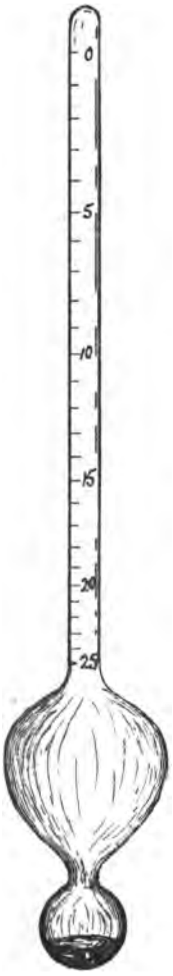

FIG. 66.

The principle involved in the use of such hydrometers will be readily understood. Suppose the instrument to be floated in water so that the bulbs and part of the stem are immersed, and let the position of the water-surface upon the stem be observed. Then let the hydrometer be dried and placed in some other liquid. If the instrument sinks so that the surface of this liquid rises beyond the mark on the stem where the water stood, it is manifest that this second liquid has a specific gravity less than 1. Similarly, if the water-mark stands above the surface of the second liquid, it follows that this liquid has a specific gravity greater than 1. If now the stem be divided by a number of marks and the position of the surface of liquids of known specific gravities upon it when the hydrometer is floated in them be noted, it is clearly possible to draw up a table showing the specific gravity which corresponds with each mark upon the stem. This is the plan adopted in what is known as Baume's hydrometer. Equidistant marks upon the stem are numbered consecutively, and a table is supplied of the specific gravities which correspond to each reading of the scale.

It is also possible to make the marks by observing the depth of immersion in various liquids of known density, so that the density in question can be read off directly from the scale upon the stem. This is the method used in the Twaddell's hydrometer. Finally, if the same hydrometer were to be used for liquids less dense and for liquids more dense than water the stem would have to be inconveniently long. Hence it is customary to use at least two, one for liquids less dense than water, which sinks only just above the bulbs in water and is graduated upwards from the water-mark, and the other for liquids more dense than water, which sinks in water nearly to the top of the stem and is graduated downwards from the water-mark. For more delicate work the number of separate instruments may evidently be increased at will, it being only necessary to use a very narrow stem, of which, clearly, more will need to be immersed, for a given difference of specific gravity, than of a wide stem.

The Twaddell's hydrometer shown in *Fig.* 66 is intended for use with liquids denser than water. The numbers on the scale in this case are arbitrarily so arranged that when placed in pure water the hydrometer sinks to the mark 0. The numbers of the other marks are to be multiplied by 0.005 and the product added to the specific gravity of water (1.000) to give the specific gravity of another liquid. . Thus if the instrument sank to the mark 20, the specific gravity of the liquid would be 1.1.

Many other methods of graduation are, as mentioned above, in use, and the student must make sure before attempting to use a hydrometer of variable immersion that he understands clearly which system has been adopted in marking it. For further information on these hydrometers he must consult larger works.

CHAPTER XI

GASES

§ 150. GASES AS A PARTICULAR KIND OF FLUIDS.—We saw, in §§ 91 and 114, that it is possible clearly to distinguish two classes of bodies which come under our observation, *solids* and *fluids*. In §§ 115-149 we have considered some of the simpler properties of a variety of fluids called *liquids*. But there are fluids which, while in some properties they closely resemble liquids, differ sharply from them in others. These are what are familiarly known as *gases*. Just as we have considered water as a typical liquid so we may, for convenience sake, think of the air which surrounds us as a typical gas. The study of air has been pursued, of course, from the most remote antiquity, but the idea that a class of substances exists having many properties in common with it is, comparatively speaking, modern, having been introduced about three hundred years ago. It was a Dutch man of science, van Helmont by name, who called attention to the chief property which distinguishes this class of fluids from liquids, and proposed the name "gas" for them, "because," he said, "they cannot be prisoned in vessels." What van Helmont meant was that, whereas if a liquid be placed in a dish or other vessel it may fill it only to a certain extent and expose a definite surface to the air above, a gas under similar circumstances begins instantly to escape from the vessel and to spread itself through the surrounding air. It is said that the name gas is connected with, and closely related to, the Dutch word for "ghost," and if so, it is not hard to see why van Helmont chose it.

From the beginning of the seventeenth century began the study of gases as a distinct variety of fluids, but it was not until the middle of that century that the property in which they most closely resemble liquids was definitely established. In 1643 an Italian, named Torricelli showed that the layer of gas which we call the atmosphere exercises a downthrust upon the surface of the earth, comparable with that which, as we have seen in §§ 133-135, is exercised by a layer of liquid upon the base of its containing vessel, and devised a means of measuring this "atmospheric pressure."

§ 151. ATMOSPHERIC PRESSURE.—That objects upon the earth's surface are subject to a pressure due to the weight of the column of air above them is a conclusion which follows from many well-known and simple experiments. One of these we have already performed several times, in the filling of a pipette and in raising the liquids in the tubes of Hare's apparatus (§ 138), as well as in the filling of the pyknometer (§ 126). If a tube, open at both ends, be placed with one end below the surface of a liquid, then the liquid (except for the effect of capillarity (§ 118) which, in any but very narrow tubes, may be neglected) stands at the same level inside and outside the tube. But if now, by sucking at the open end, we diminish the pressure of the air on the inside surface, the liquid rises, and this rise we recognise as a natural consequence of the pressure of the atmosphere upon the surface of the liquid outside the tube.

A striking experiment by which the existence of atmospheric pressure can be shown, and which also gives some idea how considerable this pressure is, may be performed as follows:—Procure a small tin can such as is often used by workmen and others to hold cold tea (or other liquors) and select for it a tightly-fitting cork. Then put about 50 or 100 *c.cs.* of water into it, place it upon a tripod over a Bunsen burner, and boil the water vigorously for some minutes. The escaping steam will gradually displace the air from within the can, and if now, before all the water has boiled away, the cork be thrust in and the burner at the same moment put out, practically nothing but water and steam will be enclosed in the can. As it cools the steam condenses, and since the water produced occupies a volume much less than that of the steam (roughly speaking 1650 *c.cs.* of steam condense to 1 *c.c.* of water) the can will, when cold, be empty save for a little water. The pressure of the atmosphere now acts only on the outside of the can, and unless the latter either leaks or is remarkably strong, it will be seen gradually to collapse, being crushed in as though a heavy weight had been placed upon it, which, indeed, as we shall see shortly, is what has really happened.

We may admit then that the atmosphere does press upon objects on the earth's surface, and, from the result of the experiment just quoted, we shall be prepared to find that it does so with great force.

We have now to discover some means of measuring this pressure, and for this purpose we shall repeat the experiment first made by Torricelli in 1643.

§ 152. THE BAROMETER.—This experiment consists simply in taking a glass tube sealed up at one end and open at the other and filling it completely with mercury. Then while the open end is closely covered with the thumb, so as to prevent escape of mercury and entrance of air, the tube is inverted, the open end placed below the surface of more mercury contained in an open dish, and the thumb withdrawn. It is found that the mercury in the tube does not run out, as might at first sight be expected, till the levels of the mercury inside and outside the tube are the same, but that a column of mercury remains standing in the tube. Torricelli found that if the tube be shorter than about 76 *cms.* the mercury continues completely to fill it, while if the tube be longer than this some of the mercury does run out, leaving an empty space, or vacuum, at the top of the tube. This space, which contains only a minute quantity of mercury vapour, has been known ever since as a "Torricellian vacuum."

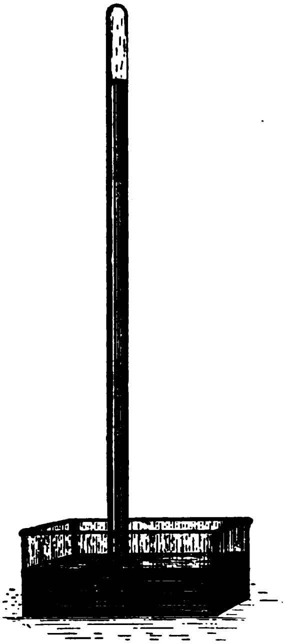

FIG. 67.

These observations led Torricelli to recognise the existence of a definite atmospheric pressure, great enough to counterbalance the pressure due to a column of mercury of a certain height, and unable to support a column of mercury longer than that.

The arrangement here described evidently affords a means of measuring the pressure of the atmosphere, or, to put it in other words, the weight of the air. It is therefore called a *barometer*, a name derived from two Greek words, meaning respectively "*heavy*" and "*measure.*"

We may now, taking advantage of the discovery of Torricelli concerning the length of the column of mercury which the atmosphere will support, proceed to construct a simple barometer.

Exercise LXII.

To set up a simple form of barometer.

Apparatus.—Thick-walled glass tube of narrow bore (about 5 *mms.*) and at least 80 *cms.* long, having one end sealed and the other ground smooth, small porcelain trough (or mortar), retort-stand and clamp, small piece of cloth or felt, small funnel, shallow wooden tray with raised edges, metre scale, filtered mercury.

[*Note.*—Remember the caution given under Exercise XLIV. § 131. To ensure its being clean mercury should be filtered before use. This is done by folding a piece of clean blotting-paper so as to form a hollow cone that will fit an ordinary glass funnel. Near the apex of this cone a small clean hole is pierced with a needle. On pouring in the mercury it runs in a thin stream through this fine hole and is collected in a clean dry vessel, any dust, etc., remaining upon the blotting-paper.]

Method.—See that the tube is clean and dry (it should be several times wiped out with small clean pieces of dry cotton wool, which may be fixed to the end of a piece of clean iron wire, or, better, held at the end of a piece of quill-tubing by the loop of a piece of glazed thread which is doubled and passed up the bore of the tube). Fix it vertically in the clamp of the retort-stand with the open end upwards and the closed end resting upon the cloth or felt folded so as to form a pad and protect the tube from contact with the iron base. With the help of the funnel very gradually pour mercury into the tube and fill it to within 2 or 3 *cms.* of the top. Then place the right thumb over the open end, loosen the tube from the clamp, and, by inclining it very gently, pass the bubble of air enclosed several times up and down the entire length. This will collect the small bubbles which form between the mercury and the glass. Replace the tube in the clamp and fill it completely. Then place mercury in the small trough or mortar to a depth of about 4 *cms.* Next grasp the tube near the top with the right hand, put the right thumb firmly over the end, pressing it down so as to squeeze out a little mercury, loosen the clamp and invert the tube. Keeping the end still firmly closed with the thumb lower it beneath the surface of the mercury in the trough or mortar, and then gently remove the thumb.

The mercury is seen to fall some *cms.* in the tube, and then to become stationary.

The general arrangement is illustrated in *Fig.* 67 (p. 196), but a large cistern is there drawn for the sake of clearness instead of the small trough or mortar actually used.

It only remains to measure, with the metre scale, the height of the surface in the tube above that in the cistern. This can easily be done correctly to 1 *mm.* by holding the scale alongside the barometer tube with its end just touching the surface of the mercury in the cistern—a position easily obtained by noting when the end and its image in the mercury just meet.

From the results of §§ 136 and 137 it follows that a knowledge of this height, combined with the knowledge that the density of mercury is 13.59 *grms.* per *c.c.*, will enable the pressure of the air in *grms.-wt.* per *sq. cm.* to be calculated. [Explain fully how this is to be done, and make the calculation from your measurements.]

> [*Note.*—The correctness of the result evidently depends upon the absence of air from the space above the mercury. To make sure that this condition is fulfilled it is only necessary very gently to incline the tube, taking care to keep its open end well below the surface of the mercury in the cistern, until the closed end is at a smaller vertical distance above that surface than the height of the mercury column just observed. If there be a true vacuum at the top the mercury will then completely fill the tube. Why? Explain fully.]

The student will be shown, in § 159 below, how the measurement of the height of the barometer column may be made much more accurately. Meanwhile, he should make sure that he fully understands the principle of the simple instrument just constructed.

PROBLEMS:—

(32) Will the diameter of the bore of a barometer-tube make any difference to the height of the column of mercury which the atmosphere can support at a given time and place? Give full reasons for your answer, and illustrate what you say by a diagram (compare §§ 133-138).

(33) Calculate to the nearest whole number the pressure of the atmosphere, in *grms.-wt.* per *sq. cm.*, when the mercury column of the barometer is 73.58 *cms.* long.

It will be manifest that if a liquid of different density from mercury be used the barometer column will stand at a different height. Knowing the density of a liquid we can easily, from the results obtained with balancing columns (that the height is inversely proportional to the density), calculate the length of the column of any other liquid which will be supported by the pressure which supports a given column of mercury.

PROBLEMS:—

(34) Assuming the density of glycerin to be 1.27 *grms.* per *c.c.*, calculate the height of the glycerin-barometer when that of the mercury-barometer is 75 *cms.*

(35) What would be the height, in *ft.*, of the mercury-barometer when a water-barometer stands at 32.8 *ft.*? (work to one place of decimals only).

§ 153. PRESSURE OF A GAS.—We have already considered, and seen how to measure approximately, one property in which air resembles other fluids with which we have already dealt, namely, the downthrust—due to the weight of the air—upon the surface on which it rests.

We have now to consider another property, and one which specially distinguishes it from the class of fluids called liquids.

As we have seen, the fact that a gas does not remain quietly in an open vessel but escapes rapidly and spreads itself through the surrounding air was noticed long since. There are many familiar examples of it. Thus some gases have strong smells, by which their presence is easily recognised, and if a vessel containing a small quantity of one of them, such as ammonia gas, be left open in a large room, it is found that in a little while the smell of the gas is perceptible all over the room.

It is also a fact that has long been known that if a small quantity of a gas be introduced into a large empty space it does not, as a liquid would, fill only a portion of that space, but spreads itself uniformly through the whole of it. It is possible to remove the air from within a large globe by means of an air-pump (§ 158) and then to introduce a small quantity of a gas and show that it spreads itself over the entire space. Thus some gases are coloured—nitrogen peroxide, for example, having a red colour, and if very little nitrogen peroxide be introduced into a large vacuous globe the red colour at once becomes visible all over that globe.

We recognise then, from these and very many similar observations, that a characteristic property of gases is to expand and fill whatever space is placed at their disposal. We shall now see that this property carries with it another, namely, that gases press outwards, downwards, and upwards against the walls of any vessel in which they may be contained. This too is a fact with which we are not unfamiliar. Every one who has ever inflated a football or "blown up" the pneumatic tyre of a bicycle knows very well that if air be thrust into a closed vessel it presses against the walls, and, if possible, forces them outwards, and knows too that the more air is forced in the stronger this pressure upon the walls becomes.

This pressure of gases is manifestly something perfectly distinct from the downthrust due to their weight, of which we have taken an example in § 151. It acts in all directions, and is a question of the volume, not merely of the depth, of the gas.

Since we now possess convenient means of measuring pressure we may proceed to make this property a subject of measurement.

§ 154. VOLUME AND PRESSURE OF A GIVEN MASS OF GAS.—The familiar illustrations of the existence of gaseous pressure which we have chosen above both refer to air which is, as we commonly say, compressed, but any quantity of air enclosed within a vessel will also exercise pressure on its walls. The simple apparatus used in Exercise LXII. will serve to show this.

Exercise LXIII.

To show that a small quantity of air enclosed in a considerable space exercises pressure upon the walls of the containing vessel.

Apparatus.—Barometer set up in Exercise LXII.

Method.—Incline the barometer tube till the mercury reaches the top, place the thumb over the end, remove from the cistern and invert. Then empty out a little of the mercury so that it stands, say, 1 *cm.* from the open end, again close that end with the thumb, invert the tube, place the open end under the mercury in the cistern and cautiously remove the thumb. Measure the height of the mercury in the tube above that in the cistern. How does it compare with the height of the barometer column? What conclusion can you draw?

The question now naturally suggests itself upon what the magnitude of this pressure depends in any given case. It is natural, from what we have already seen, to suppose that there will be some connection between the volume into which a definite amount of air is admitted and the pressure of that air. We may put this supposition to the test by taking two barometers with tubes of the same bore, but of unequal lengths, and introducing into the vacua equal amounts of air. It is easy to see, and can be readily proved by experiment, that the height of the mercury column in each when it is made into a true barometer will be the same, and hence that the vacuous portion in the longer tube will have a greater volume than that in the shorter tube. Now suppose the two tubes to be treated as in Exercise LXIII., and let each be filled with mercury, not completely, but to within the same distance of the open end. Then we shall obviously include, on inverting the tube, approximately equal quantities of air, and one of these equal quantities will be introduced into a larger space than the other. Then by comparing the actual heights of the mercury columns with one another and with that of the barometer we can see whether there is any difference in the pressures.

Exercise LXIV.

To compare the pressures exercised by (approximately) equal quantities of air when introduced into two unequal vacuous spaces.

Apparatus.—As for Exercise LXII., with additional barometer tube of the same bore, but about 100 *cms.* long.

Method.—Fill both tubes with mercury, as in Exercise LXII., so that the liquid stands an equal distance, say, 1 to 2 *cms.* below the open end of each. Invert each tube and place it upright in the cistern with the precautions already mentioned. With the two tubes side by side compare the heights of the columns of mercury. Do you notice any difference? Confirm, or correct, your impression by careful measurements with the metre scale.

Do you agree that the same quantity of air exercises less pressure when it occupies a larger volume?

Having now satisfied ourselves that there is some connection between the pressure of a given quantity of air and the volume occupied by it, and having gained a rough idea of how the one depends upon the other, we have next to determine accurately the relation between them.

To do this we must have some means of altering at will, say, the volume of a definite mass of air and measuring the accompanying alteration of pressure.

For this purpose the apparatus shown in *Fig.* 68 will be found convenient.

It consists of two glass tubes, *a* and *b*, about 40 *cms.* long, joined by a piece of *thick-walled* rubber tubing, *c*, of considerable length and securely bound to one end of each. One of these tubes, *a*, is sealed at one end, and is graduated so that the volume

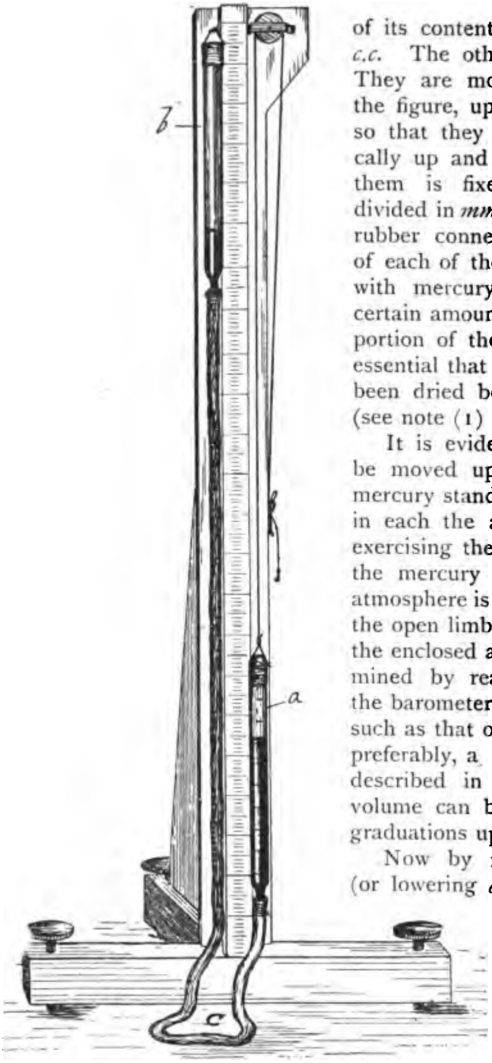

FIG. 68.

of its contents can be read to 0.1 *c.c.* The other tube, *b*, is open. They are mounted, as shown in the figure, upon a wooden stand, so that they can be moved vertically up and down, and between them is fixed a vertical scale divided in *mms.* The whole of the rubber connecting tube and part of each of the glass tubes is filled with mercury, which encloses a certain amount of air in the upper portion of the closed tube. It is essential that this air should have been dried before being enclosed (see note (1) below).

It is evident that if the tubes be moved up or down until the mercury stands at the same level in each the air enclosed in *a* is exercising the same pressure upon the mercury in that limb as the atmosphere is exerting upon that in the open limb, *b*. The pressure of the enclosed air can then be determined by reading the height of the barometer, either a simple one such as that of Exercise LXII., or, preferably, a "Fortin," such as is described in § 159 below. Its volume can be read off from the graduations upon the tube.

Now by raising the tube *b* (or lowering *a*) the volume of the enclosed air can be diminished, the pressure at the same time becoming greater, as is shown by the fact that the mercury in *b* stands at a higher level than that in *a*. The difference of level in the two limbs can be easily determined upon the vertical scale with the aid of the reading-telescope (§ 139), and it will be plain that, to find the pressure of the enclosed air, this height must be *added to* the barometer-reading. Similarly, by raising *a* (or lowering *b*) the volume of the air enclosed may be increased, and it is then seen that the mercury in *b* stands below that in *a*. In other words, the pressure of the enclosed air is now less than that of the atmosphere. The difference in level can be determined as before, and this height must now be *subtracted from* the barometer-reading to give the pressure.

It is thus possible to obtain as many pairs of readings as may be desired of the volumes (in *c.cs.*) and corresponding pressures (in *cms.* or *mms.* of mercury—see § 136) of a given mass of air. It is necessary, for a reason which will be explained in § 155, that, for these readings to be comparable with one another, the temperature of the enclosed air should be the same during all the measurements, and if the changes of volume are effected slowly, and a little time is allowed to elapse before each measurement is taken, it may be assumed that the temperature remains equal to that of the laboratory as indicated by an ordinary thermometer. If the temperature of the laboratory changes appreciably during the course of the experiment the numbers obtained will be comparatively worthless.

Exercise LXV.

To determine the relation between the volume and pressure of a given mass of air at a constant temperature.

Apparatus.—Reading-telescope, barometer, apparatus of *Fig.* 68.

Method.—Take a careful reading of the barometer (best as directed in § 159). Determine the difference of level of the mercury in the two limbs and the corresponding volume of the enclosed air for several different positions of the tubes *a* and *b* (at least six measurements should be taken, three with the mercury in the open limb higher than that in the closed limb, and three with it lower). Remember that if the level in *a* is above that in *b* the difference must be subtracted from the barometer reading to give the pressure, while if the level in *b* is above that in *a* the difference must be added. Record the volumes and corresponding pressures in parallel vertical columns. Find the products of the corresponding pairs—pressure × volume. What relation can you recognise between these products?

The experiment just performed was first made in a slightly different way by a famous Irishman, the Hon. Robert Boyle, in 1662. Making use of an apparatus, the principle of which is the same as that of the one just used, he showed that the pressure of a given mass of air is, at one and the same temperature, inversely proportional to its volume. This rule has since been found to hold good for gases in general (see, however, Note (2) below). It is called "*Boyle's Law*," and may be stated in other words as follows :—The product obtained by multiplying the measures of the volume and pressure of a given mass of a gas is constant at the same temperature; or in symbols, if the pressure of a given mass of gas be p units (say, *mms.* of mercury), and its volume be v units (say *c.cs.*), then, whatever value we may choose to give to one of these quantities, the other always acquires such a corresponding value that

$$pv = \text{a constant.}$$

It will easily be seen that a knowledge of this "law" will enable us to say what will be the effect, for example, of a given change of pressure upon the volume of a certain mass of gas.

PROBLEM :—

(36) A mass of 2 *grms.* of hydrogen has a volume of 22.5 *litres* when the barometer stands at 76 *cms.* What will be its volume when the barometer is at 75 *cms.*? At what pressure (in *cms.* of mercury) will its volume be 20 litres?

[*Notes.*—(1) The air enclosed in the limb *a* must be *dried*, because the vapour of water does not obey Boyle's Law at ordinary temperatures. When its volume is diminished its pressure does not increase rapidly enough to keep the product constant, and if its volume is made small enough it condenses to a liquid. The dried air may be introduced into the tube as follows :—The tube *a* is disconnected from the rubber, completely filled with mercury, and inverted with its open end under mercury (as in Exercise LXII.). Then the dry air is bubbled up into it, from a fine tube turned up below the open end, till it is about half filled. This air may be supplied from a pair of rubber bulbs like those sold with scent sprays, and is dried by causing it to bubble slowly through a little strong sulphuric acid in a wash-bottle, and then to pass through a U-tube of dried calcium chloride on the way to the fine delivery tube. Then the rubber tube *c* is stopped at the open end with a glass plug, mercury is poured into *b*, the plug is slightly loosened so that this mercury runs through and fills the rubber tube, and the latter is again firmly closed. A tightly-fitting cork is next placed in the open end of *b*, the stopped end of the rubber tube is held below the surface of the mercury and the plug removed there, and it is then slipped, still under mercury, over the open end of *a* and made fast.

(2) Very careful measurements have shown that Boyle's Law does not really express the actual behaviour either of air or of any other known gas. The product, pressure × volume, is not absolutely constant, though so long as the volume is neither very small nor very great (and the pressure therefore neither very great nor very small, as the case may be) the deviations from it are extremely slight, and are not perceptible with such an apparatus as we have used. Moreover, Boyle's Law does not, as hinted in Note (1), apply to those gases, usually called "vapours," which, when their volume is sufficiently diminished at a particular temperature, change into liquids. These vapours will be studied further, when the subject of heat comes to be considered, and it will then be seen that the temperature at which it is observed has a great deal to do with the question whether a particular gas will follow, more or less nearly, the rule called Boyle's Law.]

§ 155. EFFECT OF CHANGE OF TEMPERATURE UPON THE VOLUME AND PRESSURE OF GASES.—It has been mentioned several times already—compare §§ 5 and 124—that changes of temperature affect the extension of solids and liquids. In the case of solids the effect is small, and, except in cases where great accuracy is required, as in the comparison of standards, may be neglected. In the case of liquids the effect of heating or cooling is much greater, and it is often necessary to take it into account, although here too, for practical purposes, it may, to a large extent, be ignored.

But it is far otherwise with gases. Changes of temperature cause, in this case, changes of volume which, compared with those occurring in liquids, are very great, so large indeed as to be well within the range of even rough measurements, and far too large therefore to be ignored in any case.

We cannot here enter upon an experimental study of these changes—that will belong to the subject of Heat—but must content ourselves with the statement of certain important results.

When heat is supplied to a given mass of any gas it is found that one of two things happens. If the pressure of the gas is allowed to remain the same (for example, if the gas is enclosed in a vessel one of whose walls is movable and is kept in position by the pressure of the atmosphere) then the volume of the gas increases. If, on the other hand the gas is rigidly confined so that its volume cannot alter, then the pressure of the gas increases. In the former case the gas is said to be heated at constant pressure; in the latter case it is said to be heated at constant volume.

It is of course possible that both the pressure and the volume may be allowed to alter. In that case, on supplying heat, the volume is increased and so is the pressure, but the increase of volume is not so great as it would be had the pressure remained the same, and the increase of pressure is less than that which would have occurred had the volume remained unaltered.

Careful experiments have shown that these changes of volume and pressure follow an exceedingly simple rule. To understand this rule the student must be familiar with certain terms used in the science of Heat. If an ordinary "Centigrade" thermometer be examined it will be seen to consist of a glass tube of very fine bore, having a bulb at one end and being sealed up at the other. The bulb and part of the tube are filled with mercury. When the instrument is warmed the glass and the mercury both increase in volume, but the mercury increases more than the glass and so rises in the stem. There are marks or graduations upon this stem, one of which is numbered 0 and another 100. These are respectively the points to which the mercury falls when the instrument is placed in melting ice, and rises when it is placed in the steam from water boiling under certain conditions. The distance between these two points is divided into 100 parts called "degrees." Equal divisions or degrees are marked off above the 100 and below the 0, the latter being distinguished as -1, -27, etc.

Suppose now this instrument were placed in a certain gas and registered, say, 15, and then heat were supplied to that gas until the mercury in the tube had risen to the mark numbered 16, we should say that the temperature of the gas had been raised from 15° C. to 16° C., or that the gas had been *heated through* 1° *C.*

The rule or rules which express the effect of rise of temperature upon the volume or pressure, or both, of a given mass of any gas may now be stated as follows:—

The volume of a given mass of any gas increases by $\frac{1}{273}$ *part of its volume at* 0° *C. for every* 1° *C. through which it is heated at constant pressure.*

This rule was discovered by a Frenchman named Gay-Lussac, but in England it is usually called the "Law of Charles."

Its meaning can be easily illustrated as follows. Suppose we had 273 *c.cs.* of air at 0° C. If its temperature were raised from 0° to 1° C. its volume would increase by $\frac{1}{273}$ of 273 *c.cs.*, that is by 1 *c.c.*, and would become 274 *c.cs.*, the pressure of the air remaining the same. Similarly, if any volume, say, v *c.cs.* of air at 0° C. were heated till its temperature reached 1° C., its volume would increase by $\frac{v}{273}$ *c.cs.* and become $\left(v+\frac{v}{273}\right)$ *c.cs.* If it were heated through 2° C. it would increase by $\frac{2}{273}$ of v *c.cs.* and become $\left(v+\frac{2v}{273}\right)$ *c.cs.* Finally, if it were heated through any number of degrees, say $t°$ C., it would increase by $\frac{t}{273}$ of v *c.cs.* and become $\left(v+\frac{vt}{273}\right)$ *c.cs.*, the pressure, as before, remaining constant.

This quantity $\left(v+\frac{vt}{273}\right)$ *c.cs.* can be more conveniently written $v\left(1+\frac{t}{273}\right)$ *c.cs.* or, better still, $v \times \frac{273+t}{273}$. Then the Law of Charles may be shortly expressed thus : —

$$\text{Volume at } t° \text{ C.} = \text{volume at } 0° \text{ C.} \times \frac{273+t}{273}.$$

PROBLEM :—

(37) A certain mass of a gas, having a certain pressure, has a volume of 91 *c.cs.* at 0° C. What will be its volume at 27° C., its pressure remaining the same?

Similarly, Gay-Lussac found that the volume of a given mass of gas diminishes by $\frac{1}{273}$ part of its volume at 0° C. for every 1°C. through which it is cooled, its pressure remaining the same. By reasoning exactly similar to that used above we can easily see that if v *c.cs.* of a gas at 0° C. be cooled, at constant pressure, to $-t°$ C., the volume will become $\left(v-\frac{vt}{273}\right)$ *c.cs.*—that is to say, as before, $v \times \frac{273-t}{273}$ *c.cs.*

In short,

$$\text{Volume at } -t° \text{ C.} = \text{volume at } 0° \text{ C} \times \frac{273-t}{273}.$$

PROBLEM :—

(38) A certain mass of gas has a volume of 21 *cc.s.* at 0° C. The temperature is lowered to $-13°$ C., the pressure of the gas remaining the same. What will now be its volume?

We can now apply this "law of Charles" generally. If a gas have a volume of v *c.cs.* at any temperature $t°$ C., and this temperature be changed, at constant pressure, to some other, say $n°$ C., we can calculate the change of volume, which will occur as follows :—

Suppose the gas, in the first place, cooled down from $t°$ C. to 0° C. As we have just seen—

$$\text{Volume at } t° \text{ C.} = \text{volume at } 0° \text{ C.} \times \frac{273+t}{273};$$

whence

$$\text{Volume at } 0° \text{ C.} = \text{volume at } t° \text{ C.} \times \frac{273}{273+t} = v \times \frac{273}{273+t} \textit{ c.cs.}$$

Then let this volume of gas be brought from 0° C. to $n°$ C. We have

$$\text{Volume at } n° \text{ C.} = \text{volume at } 0° \text{ C.} \times \frac{273+n}{273}$$

$$= v \times \frac{273}{273+t} \times \frac{273+n}{273} \textit{ c.cs.}$$

$$= v \times \frac{273+n}{273+t};$$

or, to sum up,

$$\text{Volume at } n° \text{ C.} = \text{volume at } t° \text{ C.} \times \frac{273+n}{273+t}.$$

PROBLEM :—

(39) A certain mass of gas has a volume of 145 *c.cs.* at 17° C. What will this become when the temperature is changed, at constant pressure, (1) to 5° C., (2) to $-3°$ C.?

The variation of pressure with temperature, the volume remaining constant, was studied by another Frenchman named Regnault. He found that a precisely similar rule holds good.

The pressure of a given mass of any gas increases by $\frac{1}{273}$ *part of its value at* 0° *C. for every* 1° *C. that its temperature is raised,* *and decreases by $\frac{1}{273}$ of its value at* 0° *C. for every* 1° *C. that its temperature is lowered, the volume being kept constant.*

The student should have no difficulty in formulating the relations between pressure and temperature, at constant volume, in the same way as those between volume and temperature, at constant pressure, have been formulated above.

When he has done so he will be able easily to solve the following problems :—

(40) A certain mass of gas has a pressure of 819 *mms.* of mercury at 0° C. What will this become if the temperature be (1) lowered, at constant volume, to − 23° C., (2) raised, at constant volume, to 7° C. ?

(41) Calculate the change of pressure produced when a certain mass of gas whose pressure at − 3° C. is equal to that of 54 *cms.* of mercury is heated, without change of volume, to 102° C.

§ 156. CORRECTION OF VOLUME OF GAS FOR TEMPERATURE AND PRESSURE.—From the results established in § 154, and from the facts quoted in § 155, it will be plain that such an expression as "the volume of a gramme of air" is quite without definite meaning unless it be stated, firstly, what is the pressure of that air, and, secondly, what is its temperature. Thus it might be stated that the volume of 1 *grm.* of hydrogen gas is, very nearly, 11.2 *litres*, but this is only true under certain conditions. It is true when the hydrogen has a pressure of 760 *mms.* (of mercury) and a temperature of 0° C. It is also true if the pressure be 824 *mms.* and the temperature 23° C., or the pressure 712.7 *mms.* and the temperature − 17° C., and so on, there being obviously any number of pairs of temperatures and pressures for which the statement is true.

It would evidently be convenient if some standard temperature and standard pressure could be chosen, with the understanding that statements of volume, not otherwise qualified, referred to them. This is actually done. *The standard temperature is* 0° *C., and the standard pressure* 760 *mms. of mercury.* These are often spoken of as "*normal temperature and pressure,*" a phrase which is sometimes abbreviated to "N.T.P."

It is evidently necessary in view of this to be able, when we have measured the volume of a quantity of gas at a certain temperature and pressure, to calculate what that volume would become at 0° C. and 760 *mms.* This is an operation which has constantly to be performed in the study of chemistry where some gas or other is frequently produced by the action of one substance upon another, and it is required to specify the volume produced from definite masses.

The alteration of volume which accompanies a given alteration of *pressure* is easily found by applying Boyle's Law (§ 154), and that due to a given alteration of temperature can be determined from the Law of Charles (§ 155). For convenience sake, however, it is desirable, when both pressure and temperature are altered together, to make the calculation in one step. This is done by taking advantage of the fact, experimentally proved by Regnault, that the *product of the volume and pressure varies* when the temperature is changed in the same way as either of them singly when the other is kept constant, namely, *by* $\frac{1}{273}$ *part of its value at* 0° *C. for an alteration of temperature of* 1° *C.*

Thus if the volume of a gas be v_0 *c.cs.*, and its pressure p_0 *mms.* (of mercury) when its temperature is 0° C., and v_t *c.cs.* and p_t *mms.*, its volume and pressure at t° C., we have, by the same method of reasoning as was used in § 155,

$$p_t v_t = p_0 v_0 + \frac{p_0 v_0 t}{273} = p_0 v_0 . \frac{273 + t}{273};$$

or, in words,

$$\left.\begin{array}{l}\text{the product of the pres-}\\ \text{sure and volume at } t^\circ C.\end{array}\right\} = \left\{\begin{array}{l}\text{the product of the pres-}\\ \text{sure and volume at } 0^\circ C.\end{array}\right. \times \frac{273 + t}{273}.$$

From this it follows that

$$\left.\begin{array}{l}\text{the product of the pres-}\\ \text{sure and volume at } 0^\circ C.\end{array}\right\} = \left\{\begin{array}{l}\text{the product of the pres-}\\ \text{sure and volume at } t^\circ C.\end{array}\right. \times \frac{273}{273 + t};$$

or, in symbols,

$$p_0 v_0 = p_t v_t . \frac{273}{273 + t}.$$

We thus know the relation between the quantities which we have designated by the symbols p_0, v_0, p_t, v_t, and t, and if all but one be known, it is merely a matter of solving a simple equation to find that one.

PROBLEMS :—

(42) A certain mass of gas has a volume of 50 *c.cs.* at N.T.P. What will this become when the pressure is 750 *mms.* and the temperature 13° C. ?

(43) A quantity of gas was measured at a temperature of 27° C. and a pressure of 780 *mms.* and found to have a volume of 152 *c.cs.* What would its volume have been at 0° C. and 760 *mms.* pressure?

(44) When the volume of a certain mass of gas was 120 *c.cs.*, and its temperature 21° C., its pressure was found to be 600 *mms.* What would be its pressure when its volume was 100 *c.cs.* and its temperature 0° C.?

(45) When the pressure of a certain mass of gas was 819 *mms.* its volume was 40 *litres* at 0° C. At what temperature would its volume become 50 *litres*, the pressure being then 744 *mms*?

(46) Show that if the volume of a gas at $t°$ *C.* be v_t *c.cs.*, and its pressure p_t *mms.*, and its volume at $n°$ *C.* be v_n *c.cs.*, and its pressure then p_n *mms.*,

$$p_n v_n = p_t v_t \cdot \frac{273+n}{273+t} \text{ (compare § 155).}$$

§ 157. MASS AND DENSITY OF GASES.—It will be plain that the determination of the mass of a gas must be carried out in a closed vessel by a method of weighing by difference (§ 103). In practice a glass globe, of considerable volume, is suspended from one end of the beam of a very delicate balance and counterpoised by another vessel of equal size hung from the other end. (The reason for this arrangement will be found in § 160 below.) This globe is exhausted as completely as possible by means of some form of air-pump before being finally counterpoised. Then it is filled, through a specially constructed stopcock, with the gas under examination, and the mass needed to restore equilibrium with the counterpoise is observed.

The object of determining the mass is usually to find the density of the gas, and it is evident that to do this the volume of the gas weighed must also be known. But, as we have seen, the volume of a given mass of gas is a variable quantity depending on its temperature and pressure. Hence—density being the mass of unit volume (§ 110)—the expression density of a gas is without definite meaning unless the temperature and pressure at which the unit volume in question is to be weighed are stated.

In practice the density of a gas is always taken to mean the mass of unit volume measured at 0° C. and 760 *mms.* pressure (N.T.P.).

Hence, in the actual determinations, the volume of the globe, and the precise temperature and pressure of the gas enclosed in it, must be known. The volume of the globe is calculated, with the help of a table like that in § 127, from the mass of water which it is observed to contain at various temperatures. The temperature of the gas enclosed in it is adjusted to some known value by immersing the globe, during and after the filling, in a large bath whose temperature is indicated by delicate thermometers. Its pressure is allowed to become equal to that of the atmosphere by first very slightly overfilling the globe, and then momentarily opening the stopcock after the adjustment of the temperature is completed. Then, by making an accurate reading of the barometer (as in § 159 below), the value of this pressure is determined. From the data so obtained the "corrected" volume, at 0° C. and 760 *mms.*, can be calculated, and this, with the observed mass, gives the density.

The determination of the densities of gases is one of the most delicate and difficult measurements in the whole of physics. It is quite impossible with ordinary apparatus, such as is likely to be at the student's command, to carry it out with even moderate accuracy. The values accepted for the densities of the common gases were, most of them, determined some fifty years ago by Regnault, and they have been revised and corrected during the last few years by Lord Rayleigh.

The following are the densities of some gases in *grms.* per *c.c.* at N.T.P. :—

Hydrogen, 0.0000899.	Carbon dioxide, 0.001969.
Oxygen, 0.001429.	Air (dry), 0.001293.
Nitrogen, 0.001250.	

It will be noticed that these densities are very small compared with any that are found for liquids and solids. This circumstance leads to a special arrangement respecting the specific gravities of gases.

Remembering the definition of specific gravity given in § 123, and taking—from the table of § 127—the mass of 1 *c.c.* of water at 0° as 0.999871 *grms.*, it will be seen that the specific gravity of hydrogen becomes 0.00008991 . . . . when the density of water is taken as the unit. Now such numbers are very inconvenient and cumbersome. Hence *in reckoning the specific gravities of gases the unit of density chosen is not that of water, but that of the gas, which has the smallest density, namely, hydrogen.*

Thus the specific gravity of air will be found by dividing the density of air (0.001293 *grms.* per *c.c.*) by the density of hydrogen (0.0000899 *grms.* per *c.c.*), and is therefore 14.4 (very nearly).

It is necessary carefully to bear this in mind, because in chemistry the specific gravities of gases are constantly used, and are numbers of the highest importance.

[*Note.*—The density of *air* is also very frequently adopted as a unit in reckoning the specific gravities of gases. It is evident that the specific gravities referred to air will be only $\frac{1}{14.4}$ of those referred to hydrogen.]

§ 158. THE COMMON AIR-PUMP.—In preceding sections we have more than once referred to the possibility of removing the air from the interior of a vessel by means of an air-pump, and though this is not the place to describe all the various forms which such an instrument may take, it may be useful to indicate shortly the mode of action of a common and simple form shown diagrammatically in *Fig.* 69.

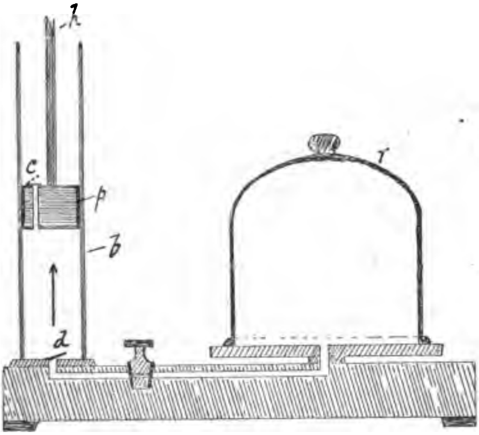

FIG. 69.

It consists essentially of two vessels connected by a tube or channel. The larger vessel in the diagram, marked *r*, is called the "*receiver*," and is commonly a bell-jar of thick glass, having its lower edge ground smooth so as to fit tightly upon a flat greased plate, from the centre of which runs the tube which communicates with the smaller vessel. This smaller vessel, *b*, is called the "*barrel*," and is simply a metallic cylinder in which a piston, *p*, fits tightly. This piston can be moved up and down by applying force to the rod *h*. In the piston is a valve, *c*, which opens upwards, and at the end of the connecting-tube, between the receiver and the barrel, is another valve, *d*, which also opens upwards.

Now suppose the piston to be at the bottom of the cylinder or barrel. When it is raised the valve *c* closes, being forced down by the thrust of the atmosphere above it. The space immediately below the piston is thus left empty, and the pressure of the air in the receiver forces up the valve *d*, and thus part of the enclosed air passes into this space. This continues until the up-stroke of the piston is finished. In the diagram the piston is represented as rising. When the down-stroke of the piston begins the volume of the air in the barrel is diminished, and its pressure begins to increase, and hence the valve *d* is shut down. As the piston descends this pressure becomes greater and greater, until it exceeds the downthrust of the atmosphere, and the valve *c* opens. When the piston again reaches the bottom of the cylinder the air which was withdrawn from the receiver during the up-stroke is thus entirely expelled, and the series of operations here described is begun again.

The air-pump in practical use is not quite so simple in construction as this. The valves, for example, are not mere flaps, but consist of little hollow brass cones, which fit into conical nozzles called valve-seats. The cylinder also is often placed horizontally. But the principle of its action is precisely that given above. The plate upon which the receiver is represented as being placed can of course be screwed off the connecting-tube, and this tube can be joined up to a vessel of any other form.

It is easy to see that if we know the volume of the receiver and connecting-tube up to the valve *d* on the one hand, and of the barrel on the other hand, we can, by applying Boyle's Law (§ 154), calculate the alteration in pressure produced by any number of strokes of the piston.

Thus let the volume of the receiver, etc., be V *c.cs.*, and that of the barrel B *c.cs.* Let the piston be at the bottom of the cylinder, and suppose the pressure of the air in the receiver to be equal to that of p *mms.* of mercury. When the piston is raised to the top of the barrel the volume of the enclosed air has changed from V *c.cs.* to $(V+B)$ *c.cs.* Its pressure can be found by Boyle's Law, for if the new pressure be x *mms.*, we have, by § 154,

$$pV = x(V + B);$$

whence

$$x = p\,\frac{V}{V+B}.$$

This, then, is the pressure after one complete stroke, for the down-stroke of the piston does not alter the pressure in the receiver, but merely closes the valve *d*. At the beginning of the next up-stroke the volume of our gas is V *c.cs.*, its pressure $p\,\frac{V}{V+B}$ *mms.* The volume is again increased to $V + B$ *c.cs.*, and the pressure falls to, say, y *mms.* Then—

$$\left(p\,\frac{V}{V+B}\right)V = y\,(V + B);$$

whence

$$y = \left(p\,\frac{V}{V+B}\right)\frac{V}{V+B}$$

$$= p\left(\frac{V}{V+B}\right)^2 mms.$$

Similarly, after three strokes the pressure is $p\left(\frac{V}{V+B}\right)^3$ *mms.*, and after n strokes $p\left(\frac{V}{V+B}\right)^n$ *mms.*

PROBLEMS :—

(47) The receiver of an air-pump has a volume of 4 *cub. ft.* That of the barrel is 1 *cub. ft.* The pressure of the air in the receiver is originally equal to that of 30 *ins.* of mercury. What will it become after 4 strokes?

(48) The receiver of an air-pump had a volume of 6 *litres.* The pressure of the air in it was equal to that of 80 *cms.* of mercury. After 2 strokes this pressure sank to 45 *cms.* What was the volume of the barrel?

All the other forms of air-pump depend on the same fundamental principle, namely, that of increasing the space available for a gas, and then removing the portion of it which expands into the additional space provided.

§ 159. THE FORTIN BAROMETER.—We have seen in § 152 above the principle upon which the measurement of atmospheric pressure depends, and have constructed a simple but not very accurate form of barometer. It was pointed out that the vertical distance between the surface of the mercury in the tube and that of the mercury in the cistern gives a measure of the pressure of the atmosphere. Now the pressure, as thus determined, is found to differ considerably in different places, and even at the same place it varies at different times. These variations are shown by a rise or fall of the mercury in the tube, and since this rise or fall must be accompanied by a flow of mercury from the cistern to the tube, or from the tube to the cistern, as the case may be, it follows that the level in the cistern also changes, though not to so large an extent as that in the tube. [Why not? Explain fully.]

With the method of measurement which we adopted in § 152 this movement of the cistern-level does not matter, since the zero of our scale is always adjusted to the actual level in the cistern. But when we come to deal with a fixed barometer, kept always ready for reference, it becomes a source of serious inconvenience. The method of placing a scale alongside the tube is tedious, and the measurements so made are not very accurate. It would manifestly be preferable in such a case to have a fixed scale on which to read the height of the column. The difficulty, however, at once arises that as the level in the cistern alters the zero of a fixed scale will not move to correspond to it.

One way of avoiding this difficulty which suggests itself is that the level of the mercury in the cistern should be kept always the same. It can easily be shown that an alteration in the cistern-level does not affect the height of the barometer column (for example by pouring more mercury into the cistern of the simple barometer made in Exercise LXII. and measuring the height of the column again). This device of maintaining the mercury in the cistern at a constant level is very successfully applied in what is known, from the name of its inventor, as the Fortin Barometer. It is obviously undesirable that the quantity of mercury in the cistern should be increased or diminished, and so the adjustment of level is made by altering the capacity of the cistern itself.

The form of cistern used is shown in *Fig.* 70. It consists essentially of a short wide *glass* cylinder, *a*, which has a closely-fitting brass cover, *c*, fixed upon it by means of the screws *e*, *e*. Through a hole in this cover passes the lower end of the barometer tube, *b*. To a constricted portion of the tube just above this hole is bound firmly an annular piece of *chamois leather*, which is then folded over and bound to a boxwood collar fixed upon the brass cover. The advantage of this arrangement lies in the fact that *chamois leather* allows air to pass through it freely, but is impervious to mercury, so that the cistern, while practically open to the air, is closed so far as the entrance of dust from without or the loss of mercury from within is concerned. The lower end of the cylinder *a* is fixed by a brass collar into a brass cylinder *f*. Inside the connecting collar is a boxwood ring,

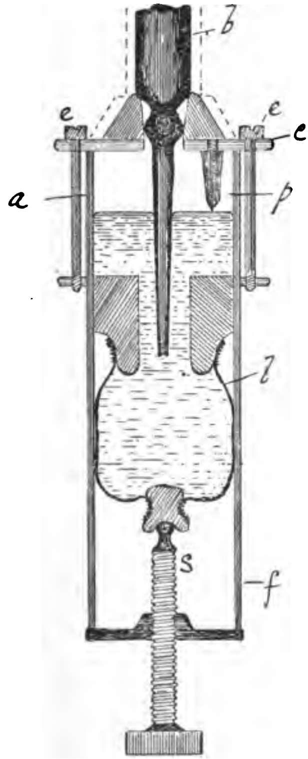

FIG. 70.

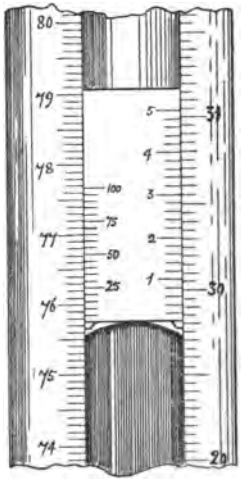

FIG. 71.

to which is firmly bound a large piece of ordinary leather, *l*, which thus forms a flexible base to the cistern. A small boxwood plug, fixed into this leather as shown in *Fig.* 70, rests upon a screw, *s*, by raising or lowering which it is evident that the capacity of the cistern—and hence the level of the mercury in it—can be altered at will. The scale upon which the height of the column is to be read is engraved upon a brass tube or sheath which encloses the glass tube of the barometer, and in which, toward the top, are two vertical rectangular slits at the back and front extending several *cms.* above and below the ordinary position of the end of the mercury column. On one side of the front slit is a scale of *mms.*, and on the other a scale of *ins.* divided in twentieths. Within the space thus left open there is a short brass cylinder, which can be raised and lowered by means of a screw (with rack and pinion), and upon which are marked two verniers, one for each scale. See *Fig.* 71.

The numbers upon the main scales give the vertical distances above the lower end of a small ivory pointer, *p*, in *Fig.* 70, which is fixed in the lid of the cistern.

If then the level of the mercury in the cistern be adjusted by means of the screw *s*, so that the point, *p*, just touches its own image in the surface the scales above will give true readings of the height.

This adjustment once made it only remains to move the short cylinder that carries the verniers till it is clear of the meniscus, look across its lower edge so that it appears as a single straight line (thus avoiding parallax, § 13), and gradually lower it until this straight edge forms a tangent to the curve of the meniscus, leaving two very small bright triangular openings visible at either edge (see *Fig.* 71). Then the height is read off from main scale and vernier.

On the *mm.* scale moves a No. 1 vernier, having 20 divisions, equal in all to 19 scale divisions. It therefore (see §§ 23 and 24) measures to $\frac{1}{20}$ *mm.* Each vernier graduation thus counts as 0.05 *mm.* The fifth is therefore marked 0.25, and so on. In *Fig.* 71 the coincidence is at the 13th vernier division, and the reading 757.65 *mms.*

The vernier upon the scale of *ins.* is also of the kind known as No. 1. It has 25 divisions, equal in all to 24 of the scale. It therefore measures to $\frac{1}{25}$ of $\frac{1}{20}$, that is, to $\frac{1}{500}$ or 0.002 *in.* The position of the vernier zero upon the scale is read off in *ins.* and twentieths, and expressed as *ins.* and a decimal. Then the vernier division where coincidence occurs is counted off, its number multiplied by 0.002 *in.* and added to the previous reading. In *Fig.* 71 the reading is 29.830 *ins.*

Exercise LXVI.

To determine the atmospheric pressure accurately to 0.05 *mm.*

Apparatus.—Fortin Barometer as described above.

Method.—*Lower* the mercury in the cistern by turning the screw at the bottom until its surface is clearly below the ivory point, and, very gradually, raise it again until that point just touches its own image in the surface. Then tap the outer casing of the tube near the top of the column. This should be done gently with the side of the forefinger. Next, by means of the screw at the side, raise the cylinder carrying the verniers clear of the meniscus, look across it so that the visible portions of its lower edge form apparently a straight line, and bring it down until this line forms a tangent to the meniscus, as in *Fig.* 71. Read the scales and verniers (as above). Take three independent readings in this way, and find from them the mean value (§ 17) of the atmospheric pressure (1) in *mms.*, and (2) in *ins.* of mercury.

[*Notes.*—(1) The reason for first lowering the mercury, and then bringing it up to the required level, is that there is enough adhesion between mercury and glass seriously to affect the shape of the meniscus, which is flatter when the mercury is falling, and more convex when it is rising. The tapping of the tube is intended to overcome this slight adhesion, and cause the convex rising meniscus to fall to its normal shape.

(2) The reading obtained really needs certain *corrections* (compare § 29). The instrument is adjusted for 0° C. Both the mercury column and the brass scales expand when heated, and that to different extents. So a thermometer is always attached to the column, from the reading of which the necessary correction may be computed. Further, capillary action (§ 118) causes the mercury column to be slightly shorter the smaller the diameter of the tube. Finally, there may be an *error* of the instrument, properly so called, due to faulty adjustment of the ivory pointer. In good work all these corrections must be determined and applied. For an account of the methods of doing this the student must consult larger works on Practical Physics.]

§ 160. UPTHRUST UPON OBJECTS IMMERSED IN AIR.—The results at which we arrived in §§ 140-142 [state them] are evidently applicable also to objects immersed in air or other gas. It is true that the mass of air displaced by an object of moderate volume is very small, and hence the upthrust of the displaced gas is a very small fraction of the weight of the object. We have hitherto ignored it in our measurements, and assumed that the weight of an object in air is to all intents and purposes its true weight. But if the object be of considerable volume this upthrust becomes too large to be neglected. Thus, as was mentioned in § 157, in weighing gases a globe is used which displaces a large volume of air. The weight of this displaced air may easily be greater than that of the gas introduced into the globe [explain how]. Now the weight of air displaced varies with the density of the air [how ?], and this density alters with the temperature [how? see § 155], the pressure [how? see § 154], and also with the amount of water-vapour present in the atmosphere, the presence of moisture diminishing the density. If then the globe were counterpoised, say, with brass weights, which would have a much smaller volume than the globe itself, it is manifest that its apparent mass would vary at different times. Hence the device adopted by Regnault of counterpoising the globe with another of equal external volume. The upthrusts on either side being then the same will neutralise one another. But the researches of Lord Rayleigh have shown that even this precaution is not enough. The experimental globe, when exhausted, is compressed to some extent by the weight of the atmosphere, which then rests only upon its outer surface. It thus displaces rather less air, and experiences less upthrust when vacuous than it does when filled. Hence it appears somewhat heavier, and the mass of the enclosed gas is under-estimated. A correction is therefore needed to allow for this shrinkage, which may amount with large globes to several *c.cs.*

In very careful weighing, such as has often to be performed in the study of Chemistry, it is not permissible to neglect the weight of the air displaced by even small objects. Hence the volumes of the objects weighed, and of the brass and platinum weights used, have to be determined, and the mass of air displaced by them calculated and its weight allowed for. This operation is known as "reduction of weighings to a vacuum."

But such refinements are only of use when the most delicate balances and the most careful standarised "weights" are employed. They are plainly unnecessary in measurements such as we have been making.

§ 161. REAL AND APPARENT DENSITY.—In the directions for the performance of Exercises LI. and LII. (§ 141) it was mentioned that the block of wood used must be varnished, and it was added that this precaution is taken so that the wood may not become sodden.

This introduces us to a difficulty which besets the determination of the density of many solids, namely, that they are what is commonly called porous. The matter of which they are composed does not really fill all the space occupied by the object. There is a vast number of small holes or cavities, too small to be perceived by the naked eye, which under ordinary circumstances are filled with air. The apparent density of such an object is that which is calculated from the mass and volume as it stands, the assumption being made that these air-spaces are really filled with the solid material of which the object is composed. This apparent density is obviously a very different quantity from the real density.

There are many varieties of *wood* which float readily upon water when they are dry, but sink in that liquid when they have been soaked in it for a long time, and it is not uncommon to find upon the sea-shore pieces of deal, for example, which, being thoroughly sodden, will no longer float.

There are many other instances of the same kind. Ordinary *wood-charcoal* has a density considerably greater than 1 *grm.* per *c.c.*, varying, according to the kind of wood used, and the method of preparing the charcoal from 1.5 to 1.8 *grms.* per *c.c.* Yet a piece of charcoal floats easily upon water, the fact being that it is extraordinarily porous. If a piece of such charcoal be heated for some time and then allowed to cool in a vacuum, it is found to sink at once in water, especially if the dissolved air has been expelled from that water by boiling it.

Another familiar example is afforded by *pumice-stone.* This is the solidified scum or froth which forms upon the molten rocks, or lavas, which well up from volcanoes. It is very porous, and floats upon water, but its specific gravity is considerably greater than 1. If it be reduced to fine powder it sinks in water, and so it does after long soaking. Pieces of it have been dredged up from the ocean floor hundreds of miles from any volcano. They had apparently floated from far away, carried by currents and driven by winds, until finally, becoming sodden, they sank.

A less extreme case is that of chalk. This, it is true, does not float upon water, but a piece of chalk becomes much heavier when saturated with water, and if the apparent density be found, calculating the volume, for example, from linear measurements, and taking the mass in air, a very different result is reached from that which would be obtained by the method of Exercise LIV. (§ 142).

As illustrations of the ways in which the densities of such substances may be found, we will take the cases of pumice and chalk.

Exercise XLVII.

To determine approximately the real and also the apparent density of a specimen of pumice.

Apparatus.—Specific gravity bottle, balance, box of "weights," counterpoise, distilled water which has been recently boiled and allowed to cool in a closed vessel (beware of repeating the experiment described in § 151), pestle and mortar, also bridge (Exercise LIII. § 42), beaker, fine thread, small camel's-hair brush, sinker.

Method.—(1) To determine the real density, powder a piece of the pumice finely. Find the mass of the specific gravity bottle (§ 107). Introduce some of the powdered pumice. Find mass of bottle + pumice (§ 107) and deduce mass of pumice. Fill up the bottle carefully with distilled water (with precautions as given in § 122) and find mass of bottle + pumice + water (§ 107). Deduce mass of water required with pumice. Thoroughly wash out all the powder from the bottle, fill with distilled water, and find mass of bottle + water (§ 107). Deduce mass of water which fills bottle, and from this and mass needed with pumice calculate volume of pumice (§ 130). From known mass and volume find density of pumice.

(2) To determine apparent density, take a piece of the same sample of pumice which has been *very thinly* coated with varnish and allowed to dry, and find its mass in air. Then determine its volume with the aid of the sinker, as in Exercise LVIII. (§ 146).

Exercise LXVIII.

To find, approximately, the real and also the apparent density of a piece of chalk.

Apparatus.—Balance, box of "weights," counterpoise, bridge, fine thread, distilled water, beaker, piece of chalk, clean blotting-paper.

Method.—Hang the piece of chalk to the hook of the balance and find its mass in air (§ 101). Allow it to hang in water until it is saturated, carefully remove the water adhering to the surface by touching lightly with blotting-paper, and again find the mass (§ 101). From this the volume of the pores or cavities can be found, for the increase is due to the water which has filled them. Finally, find the apparent mass of the (wet) chalk when suspended in water and deduce the total volume (§ 142). From the mass in air, and the total volume, calculate the apparent density. From the mass in air, and the total volume diminished by the volume of the water found necessary to fill the pores, calculate the real density.

[*Note.*—This second method is somewhat less accurate for obvious reasons, but with care will give a very good result. It should be remembered that the so-called "chalk" used for writing upon blackboards is often principally composed of barium sulphate. The experiment is better performed with real chalk.]

CHAPTER XII

TIME

§ 162. THE IDEA OF TIME.—In the foregoing chapters we have considered certain properties of the objects which we perceive through our senses, principally through those of sight and touch, and we have seen that two properties, called extension in space and mass respectively, belong to all such objects. That which possesses these two properties has been called matter. We have seen also that in addition to, and partly arising out of these properties, matter has many others, some of which we have considered more or less in detail, while at others of them we have only, as it were, glanced in passing. In our study of these properties we have been led frequently to mention, and in certain cases to measure, changes of form or volume produced, by the action upon portions of matter of something called force.

We have now, in closing this introductory sketch, to refer to yet another aspect of the way in which we become conscious of the existence and properties of objects, and, more particularly, of the way in which changes in material things present themselves to our minds.

Let us, for example, return to the case quoted in § 91. Imagine a thin steel bar clamped at one end so as to stand vertically, and let a force be applied horizontally to the free end. Our experience teaches us that, if the force be great enough, the bar is more or less bent. A little thought will show that it would be nearer the truth to say that the bar *becomes* bent. What we actually observe is that the position of certain parts of the bar, relatively to other parts, changes. The bar *was* straight, it *is* bent. These two states of the bar are perceived by us as following one another. And this peculiarity belongs to our perception of all changes.

We receive a complex set of impressions from which we make certain inferences, and these we sum up by saying "it is day"; we receive a different set of impressions, and express our inference from them in the form "it is night." Here again we are conscious that the sets of impressions are consecutive—that is to say, that one set is distinct from, and follows, the other.

Again, we become conscious of different states of our own bodies; we are hungry, or we are hot, or we are cold, or we are thirsty, and here too the impressions from which these states are inferred are recognised as separate impressions which follow one another.

We might multiply instances of this kind, but enough will have been said to show that here is a peculiarity of the way in which all change occurs. We express this peculiarity by saying that all change takes place *in time.*

Philosophers have disputed whether time is only a form under which the changes of things appear to us or whether things themselves exist in time. This controversy does not concern us. In physical science, at any rate, it is agreed to regard things themselves as existing, and changing, in time.

Nevertheless it is not easy to explain how we are to think of time. Time itself cannot be perceived: we judge of its existence, and we measure it, by the changes that occur *in* it. If, for example, we were conscious of nothing between one sound and another we should say that there was no interval between them, or that one immediately followed the other, although we might have slept between them. Supposing any one to persuade us that we had slept, we could only conceive of the interval of which we had been unconscious by imagining a series of perceivable events intervening between the sounds. A series of perceived events alone can serve us to estimate the lapse of time.

We are constantly finding out that our estimates of time have been mistaken, as when we say that it seems longer than it is since we got up. This implies a distinction between time as it is and our perception of it. Newton said that absolute, true, or mathematical time, *i.e.* time as it really is, must be conceived as flowing uniformly. But time by itself, as we have seen, cannot be perceived: we can only compare different series of changes *in time.* A perfect measure of time could only be furnished by a series of events (like, *e.g.*, the ticks of a clock) (i.) which ***seemed to us*** to be of equal duration and (ii.) which we never had occasion *to think of* as unequal in our efforts to explain their relation to other series of changes.

Attempts are usually made to illustrate our conception of "time" by the use of some figure of speech. Often it is compared with a flowing stream. Suppose there were a stream of water flowing under a bridge and we were to take our stand upon that bridge with two pebbles in our hands. Let us imagine that the water, as it flows below us, consists of a series of thin vertical layers, parallel to one another, stretching from bank to bank of the stream. Our two pebbles might be dropped exactly side by side so as to fall into the same one of these imaginary layers of water, and if that did not happen it is plain that one of them must fall into a layer of water before or behind that into which the other finds its way. The flow of the stream may be taken to correspond to the passing of time, the imaginary layers of water to what we call instants of time.

This illustration is exceedingly imperfect, but it does enable us to form some idea of how it may come about that, if we choose any two events, say the coronation of Queen Victoria and the opening of the Suez Canal, it appears to us necessary either that they should have happened at the same time, or that one of them should have occurred before the other.

§ 163. TIME AS A QUANTITY.—But if time can thus be conceived as flowing or passing it is evidently something which can be measured. To keep up our own illustration, more or less of the stream may have flowed past between the dropping of the first and second pebbles. The weak point, of course, is that there is no stream of time which we can see. We only know the events which by seeming to follow one another produce in our minds the notion of a flow. Hence all that we can do is to select two events and agree that between them a certain flow or lapse of time shall be said to have taken place. This is precisely what men are found to have done as far back as there is any record.

It is natural that the events chosen should have been such as occur without human interference, such, moreover, as recur frequently, such, finally, as are in themselves remarkable and impressive.

Of the changes which go on around us none fulfil these conditions better than the alternation of darkness and light. The succession of night and day has played the most important part in all systems of reckoning time.

The changes in the appearance of the moon too are sufficiently remarkable, and recur frequently enough to have afforded in all ages a means of marking off definite periods of time.

The succession and recurrence of the seasons, again, provides another series of events, the intervals between which have all along furnished means of indicating the lapse of time.

It is evidently of the highest importance that some uniform system of reckoning should be adopted, and fortunately this, as we shall see in § 164, has been done.

§ 164. UNIT OF TIME.—In Physics the unit of time used is the *Second.* This is defined with reference to the *day*, by which is here meant, not as in common speech the period of light as distinguished from the night or period of darkness, but the interval between two successive appearances of the sun, or of some particular star in a definite position in the sky. The day is divided into 24 *hours*, each hour into 60 *minutes*, and each minute into 60 *seconds.* The second is thus the eighty-six-thousand-four-hundredth part of a day.

Since then the size of a second depends upon the duration of a day it is necessary that we should have some idea how the latter period of time is, so to speak, marked off.

The observations of astronomers have led them to certain conclusions respecting the movements of the earth and of various heavenly bodies. We cannot so much as state them here in any detail. It must suffice to say, respecting the movements of the earth, that, in addition to certain minor ones, with which we are not now concerned, there are two of great importance.

On the one hand, the earth (whose shape was shortly described in § 90) has a spinning motion like that of a top. As in a top there are two points on its surface which do not spin, and these are called the north and south *poles.* An imaginary line joining these poles is called the earth's *axis*, and the spinning motion is described as the rotation of the earth about its axis.

On the other hand, quite independently of this, the earth moves as a whole round the sun in a path which has been accurately traced. Some idea of its form may be gained by sticking two drawing-pins, about 2 *cms.* apart, through a piece of paper fixed to a drawing board, tying a piece of string about

15 *cms.* long into a closed ring, slipping it over both pins, and then, by holding the point of the lead pencil closely against the inside of the string, describing a closed curve. This curve is called an ellipse. The points where the drawing-pins were placed are known as the foci of the ellipse. The earth's orbit, or path round the sun, has been discovered to be an ellipse, the sun occupying the position of one focus.

These movements together determine the apparent position at any instant of the sun and stars as seen from the earth.

The moon and the planets have special movements of their own, which also play an important part in deciding their apparent positions, and, to avoid unnecessary complications, they are not used in fixing the duration of the day.

The various heavenly bodies appear, as seen from the earth, to be situated in the surface of a great sphere, of which the earth is in the centre. It is upon this imaginary sphere that astronomers specify the positions of the stars and map the paths of the sun, planets, etc.

Now if a sphere be cut in two by a plane the section is a circle, and if the plane cut it into two equal parts the section is the largest circle which can be obtained in this way, and is called a great circle.

Considering any place on the surface of the earth it is, clearly, possible to have any number of planes we please passing through it which, when extended, will cut this imaginary sphere in great circles, but of all these planes one only can also contain the other two points called the north and south poles, and the great circle in which this particular plane cuts the imaginary sphere is called the *meridian* of the place in question.

The student must refer to works on astronomy for descriptions of the methods used in fixing the position of the meridian of any place, but admitting that this can be done its use will be evident from what follows.

The name **day** is used with one or other of two meanings.

On the one hand, it may stand for the interval which elapses between two successive passages of a particular star across the meridian of a place. This is known as a *sidereal day*, and is used in astronomical calculations.

[*Note.*—"Sidereal" is derived from the Greek word for star.]

On the other hand, it may stand for the interval between two successive passages of the sun across the meridian of a place. This is a *solar day*.

Now, so far as we are able to tell, the duration of a sidereal day is always the same. The apparent positions of the stars are affected only by the rotation of the earth about its axis. The stars are so far away that the movement of the earth in its orbit produces no appreciable parallax (§ 13) among them, and whatever proper motions they may have do not affect their apparent relative positions. The sidereal day, then, is simply the time during which the earth turns once about its axis.

But the solar day is found neither to have the same duration as the sidereal day, nor to be invariable. It is longer than the sidereal day. It corresponds to the time in which the earth executes rather more than one turn about its axis. This is easily accounted for. The movements of the earth in its orbit largely affect the apparent position of the sun, so largely that the latter appears to trace out among the stars a circular path known as the ecliptic. The apparent motion of the sun in the ecliptic is from west to east, and the earth rotates in the same direction about its axis. Hence the earth has to spin round a little more than once, in order, so to speak, to overtake the sun and bring any particular place into such a position that, as we have expressed it, the sun may be upon the meridian of that place. Moreover, the solar day is not of uniform duration. The earth in travelling round its elliptical orbit moves more rapidly at some parts than at others. Hence the apparent west-to-east movement of the sun varies, and from this and other causes the duration of the solar day is variable. By calculation, however, astronomers are able to allow for these irregularities and to determine the duration of a **mean solar day.** This is the day from which by subdivision our unit of time—the **second**—is derived.

[*Notes.*—(1) A mean solar day is approximately four minutes longer than a sidereal day, so that while a year contains, roughly speaking, 365 of the former, it contains 366 of the latter.

(2) It will be observed that our *unit of time*, the mean solar second, is another *fundamental unit* (see § 43).]

§ 165.—METHODS OF SUBDIVIDING AN INTERVAL OF TIME.—Having now gained some idea of the way in which the definite interval of time known as a (mean solar) day is marked off, we have to inquire how the subdivision of this interval is carried out.

It is evident that some change involving a series of events which occur more frequently than the passages of the sun across the meridian of a particular place must now be found, though, so long as the change is simple and easily observed, it is immaterial what it may be.

Apparently the earliest, but at the same time one of the most complex, of the changes used were those in the length and position of the shadow of an upright object. There are many and obvious objections to this device, even to its perfected form in the sun-dial. Leaving aside that it is useful only while the sun shines, we have seen above that the apparent movements of the sun, upon which the changes in the length and position of the shadow depend, are very complex. The sun-dial in fact is now a mere curiosity, and the student must refer for information about it to special works on the subject.

Another device of immemorial antiquity is the use of some form of water-clock, in which water is allowed to flow into or out of a vessel, and the lapse of time is indicated by the amount of water. Thus the Hindus appear to have floated a copper basin with a hole in it in a vessel of water, the interval between its being put in and its sinking—depending, of course, upon the size of the basin and that of the hole—being obviously adjustable. The Greeks, and after them the Romans, used a water-clock (or "clepsydra"), the principle of which was that a thin stream of water flowing into a vessel and gradually filling it, raised, as it did so, a float, the end of which moved alongside a scale divided so that twelve equal divisions were crossed between sunrise and sunset. Of course the "hours" thus measured had to be made longer in summer than in winter.

From the water-clock it was a natural and easy transition to the so-called hour-glass, in which the lapse of a definite time is marked by the beginning and end of the flow of a certain quantity of fine dry sand through a narrow neck uniting two glass bulbs. By adjusting the quantity of sand and the size of the neck it is possible thus to mark any desired interval. Thus hour-glasses, strictly so called, were used in churches in the sixteenth and seventeenth centuries, and small glasses, which mark only a few minutes, are still sold to indicate a suitable time for the boiling of an egg.

But of all the means of marking intervals of time with which we are acquainted none is so convenient nor so accurate as the observation of the swinging movements of a heavy body suspended so that it is free to move about a horizontal axis. Such a body is called a *pendulum.* This name is commonly applied to a thin rod or wire supported at its upper end and having some heavy object attached to the lower, but this is merely one—the most simple and convenient—of the forms which a pendulum may take. A rod of wood, for example, supported, and free to turn, upon a fine needle passing through one end would constitute a pendulum. In what follows we shall use the term in the narrower sense above indicated. The heavy body will be called the bob of the pendulum, and the to-and-fro movements spoken of as oscillations.

If the bob of a pendulum, when hanging at rest, be drawn slightly aside and then released, it falls back into its original position, and then passes beyond it, coming to rest again for an instant somewhere on the farther side of it, and then, falling back once more, passes through the position of rest and regains the position to which it was drawn aside. It is then said to have made a complete oscillation. As soon as one complete oscillation is finished another begins, and the movement continues until the resistance of the air or other causes render the oscillations too small to be seen. The oscillation is a complete double swing from one extreme position to the other and back again. The movement simply from either extreme position to the other is a half-oscillation.

Two important facts have been discovered respecting the oscillations of pendulums.

In the first place, the number of oscillations executed in a given time depends upon the length of the pendulum. We shall prove this and investigate the nature of the connection in Exercise LXXII. (§ 170). Meanwhile it is to be borne in mind that, by altering the length of the pendulum, the number of oscillations performed in a given time, and hence the duration of each oscillation, can be varied at will.

In the second place, provided the length of the pendulum remains the same, the time taken to perform an oscillation also remains the same, no matter whether the bob swings much or little to either side of the position of rest. The time taken to perform a large oscillation is the same as that taken to perform a small one. This rule was discovered, in 1583, by the great Italian whom we know as Galileo. He is said to have been led to the discovery by noticing the behaviour of a lamp which hung in the cathedral at Pisa. He observed that as this swung to and fro the oscillations, whether extending over a wide range or a narrow one, appeared to be executed in equal times. Experiments with other pendulums confirmed this, and the most valuable instrument for the measurement of time was thus discovered. It should be mentioned that this rule ceases to be true when the oscillations are made very large indeed, but so long as the swing is of reasonable extent the oscillations of a pendulum of given length are performed in equal times, or are *isochronous.*

[*Note.*—"Isochronous" comes from the Greek words for "equal" and "time" respectively.]

§ 166. THE STANDARD SECOND.—It is manifest that by taking advantage of these properties of a pendulum it is possible to divide an interval of time into any desired number of equal parts. We have only to choose a pendulum of such a length that it makes, say, n oscillations in the interval in question, and it follows that each oscillation will be performed in one nth of the whole time.

Now if the interval chosen were a mean solar day, and a pendulum were taken of such a length that it made exactly 86,400 oscillations during that interval, then each oscillation would take just 1 second. In actual practice it is found more convenient to select a pendulum of such a length that it would execute 43,200 oscillations in a mean solar day, the time of a complete oscillation being thus 2 seconds. This is done because the pendulum which makes a complete oscillation in 1 second is inconveniently short (something less than 25 *cms.*), and its movements are too rapid to be observed easily. On the other hand, a pendulum which makes a complete oscillation in 2 seconds is nearly 100 *cms.* long, and its oscillations are easily observed and counted. Such a pendulum is called a *seconds-pendulum.*

The length of a pendulum is reckoned from the point of suspension to the centre of the bob. The reason for thus reckoning it will be understood by the student when he has made enough progress in mechanics. Meanwhile it is important to remember the fact.

The length of a seconds-pendulum (that is to say of a pendulum which makes a complete oscillation in 2 seconds) varies at different places on the earth's surface, being least at places upon the equator, and becoming greater as the poles are approached.

The following table gives the length, in *cms.*, of the seconds-pendulum at some important places in Great Britain :—

Place.	Length of seconds-pendulum.
London (Greenwich)	99.41 *cms.*
Birmingham	99.42 ,,
Manchester	99.43 ,,
Newcastle	99.44 ,,
Edinburgh	99.45 ,,
Aberdeen	99.46 ,,

These numbers are obtained by calculations too intricate to be described here, but taking them for granted it is possible to construct a pendulum which shall execute a complete oscillation in two, and hence a half-oscillation in one second.

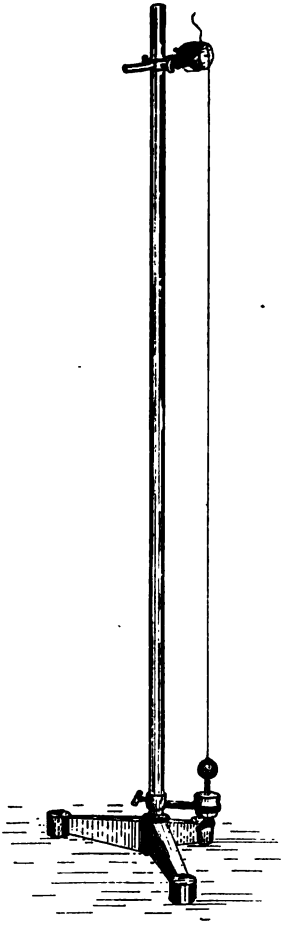

FIG. 72.

We will first describe a simple form of seconds-pendulum which the student can set up for himself. A general view of it is shown in *Fig.* 72. A tall heavy retort-stand carries two well-fitting clamps. In the upper of these is fixed, with its longer axis horizontal, a cork about 2 *cms.* in diameter. Into this cork is thrust horizontally a strong "egg-eyed" needle, so that its eye is within 1 or 2 *mms.* of the surface of the cork. The lower clamp carries another cork of about the same size with its longer axis vertical. Through this cork (which should *not* be squeezed, as is usual, before boring) a small hole is bored centrally. An ordinary cheese-headed screw slightly larger than the hole is taken, and a small piece of tinned wire (No. 12 S.W.G.) is soldered across the slot to serve as a handle, and the point is ground flat upon a stone. This is then screwed up into the hole so that the flattened point stands

0.5 *cm.* or so above the upper surface of the cork. The pendulum consists of a brass sphere having a very small wire loop soldered into two small holes in it, to which is fixed the suspending thread. This may be a piece of fine black sewing-silk, or a piece of black glazed sewing-cotton, which has been drawn several times through clean paraffin wax. The free end of this thread passes through the eye of the needle and is made fast above it, a knife-cut in the edge of the cork serving to grip the thread as it passes over. If the length be rightly adjusted this arrangement will give a very fair seconds-pendulum.

Exercise LXIX.

To set up a seconds-pendulum.

Apparatus.—Tall retort-stand (at least 1.25 *metres* high), two clamps, two corks, stout needle with small eye, screw with point ground flat and wire handle soldered across slot, brass sphere (about 2.5 *cms.* in diameter), black silk thread or waxed black glazed sewing-cotton of the size known as "No. 50," slide callipers, metre scale, straight strip of wood, at least 1.25 *metres* long, having one end carefully squared and a rectangular strip cut away on one side at the other (from about 90 *cms.* above the squared end, onwards).

Method.—Arrange the stand, clamps, corks, etc., as indicated above. Attach the brass sphere to a piece of the silk (or waxed cotton) about 1.5 *metres* long, and hang it from some convenient support so that any twist in the suspending thread may be naturally removed. Measure the diameter of the bob carefully in several directions with the slide callipers, find the mean diameter (§ 17), and deduce the radius of the sphere. Look out in the table the length of the seconds-pendulum (taking the nearest of the places given). Remember that the length of the pendulum is to be reckoned from the centre of the sphere to the eye of the needle. Hence if the length of the seconds-pendulum l *cms.*, and the radius of the bob r *cms.*, the length from the bottom of the bob to the needle's eye must be made equal to $l+r$ *cms.* Take the strip of wood and very carefully set off this length upon it, estimating to the nearest tenth of a *mm.*, starting from the squared end. Make a firm mark right across at the required distance. Then support the rod vertically, with the squared end resting upon the flattened point of the screw, so that the upper cork fits into the cut-away portion, and by moving the upper clamp bring the eye of the needle as nearly as possible upon the mark of the rod. Then fix the clamp securely, and finish the adjustment exactly by turning the screw at the bottom as may be necessary. The distance from the top of the screw to the eye of the needle should now be, within a small fraction of a *mm.*, that required. Next pass the end of the suspending-thread through the eye of the needle, draw the thread through the knife-cut in the edge of the cork, and gradually raise the bob until, as it swings, it just grazes the top of the screw. Then the pendulum is of the required length. The free end of the thread is made fast by laying it along the top of the cork and letting a small drop of sealing-wax fall upon it. It is necessary to make sure that the bob continues just to graze the top of the screw after this sealing is completed. If it does not the thread must be loosened again and readjusted.

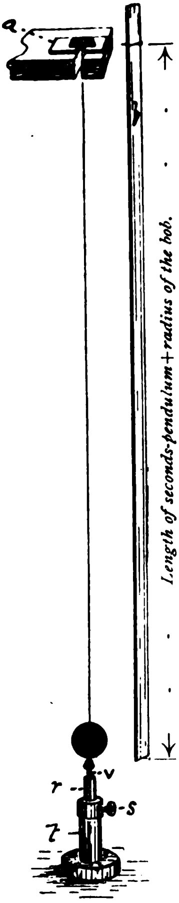

FIG. 73.

We have now provided ourselves with a pendulum of known length, the oscillations of which are executed in a known time. Once the length is fixed the lower clamp is of course loosened and the screw, etc., moved out of the way. We have next to compare the indications of this instrument with those of the instruments commonly employed for measuring time, to which we must first refer briefly.

[*Note.*—The form of seconds-pendulum here described may, with due care, be made to yield very good results, but it has many defects, which are undesirable in an instrument that is to be used for indicating the size of the standard second. Its length, for example, is not invariable, since the suspending thread will stretch in course of time. Moreover, the differing amounts of moisture present in the air at different times affect a silk or cotton thread very seriously. Hence it is far better to use a more elaborate form, which we will now proceed to describe. If possible this form should be set up once for all, and kept as the laboratory standard.

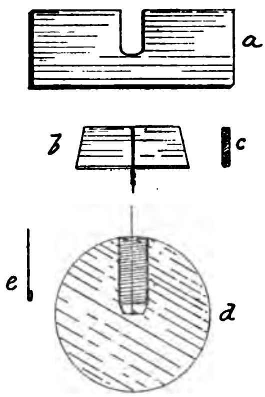

FIG. 74.

It consists of a very heavy bob, a brass sphere about 5 *cms.* in diameter, which is suspended by a fine steel wire. This wire is fixed as in the enlarged section shown in *d*, *Fig.* 74. A hole is drilled as far as the centre of the sphere, and a screw-thread is cut upon the walls of it. A brass plug is constructed of such a size that it can be screwed into and will just fill this hole. This plug is pierced with a very fine hole, and the wire is passed through this, bent up at the lower end as in *e*, *Fig.* 74, and the plug screwed home by means of a key which fits into two small holes at the top of it.

The upper end of the wire passes round a piece of clock-spring ground to an edge upon the hone. In *Fig.* 74, *b* shows a side view, and *c* an end view, of this. The clock-spring rests with the sharpened edge downwards upon a steel plate, *a*, *Fig.* 73, which is shown enlarged in *Fig.* 74, *a*. This steel plate rests upon a rigid horizontal bracket of wood screwed to the wall. The length is set off from the top of an adjustable rod of the construction shown in *Fig.* 73. *t* is a mètal tube, inside which a solid rod *r* can slide stiffly up and down, and be clamped by a screw *s*. The top of this rod carries a fine adjustment screw, *v*, upon the head of which is a small sphere. The distance $l+r$ *cms.* (the letters having the same meaning as above) is set off on both sides of a narrow flat rod of wood with a squared end (*b* in *Fig.* 73). A thin needle, with the head broken off, is thrust through at the marks, and the rod is suspended from it in the support described above. Then the adjustable rod is brought under the end, raised till the sphere on the top nearly touches the rod, and fixed by means of the screw *s*. The final adjustment is made by turning the little screw *v*. Then the steel wire is fixed round the watch-spring as in *Fig.* 74, *b*, and a length about 2 *mms.* greater than the length of the seconds-pendulum is measured from the knife-edge and cut off. The free end is passed through the plug and bent up as described. The plug is then screwed home.

On hanging it in the support the bob should just graze the top of the adjustable rod. If it be too long it must be shortened by unscrewing the plug, cutting off a tiny piece of the wire, rebending it, and proceeding as before until the right length is obtained. If by accident the wire should be made too short a fresh piece must be taken and these operations repeated.]

§ 167. CLOCKS AND WATCHES—THE STOP-WATCH.—The ordinary instruments for the measurement of time—the clock, and its modified form the watch—are in principle simply machines in which a certain amount of energy is stored, either by raising a weight or by coiling a spring, and then gradually utilised to set in motion a series of wheels which, in their turn, move two or more pointers over the surface of a divided circle or dial. Provided it is possible to ensure a uniform movement of these pointers, or hands, it is evidently merely a question of adjusting the sizes of the wheels, etc., to cause the ends of the pointers to describe a complete circle over the dial once, twice, or any desired number of times in any specified interval of time, such as a mean solar day. In practice, as is well known, these sizes are so adjusted that one of the pointers makes two complete revolutions in a mean solar day, another twenty-four, and, in cases where a third is used, this commonly makes fourteen hundred and forty. The circumference of the dial has twelve large divisions, each of which thus indicates, with respect to the most slowly moving pointer, one twenty-fourth of a day, or one hour. This pointer is hence called the "hour hand." Each of the large divisions is subdivided into five parts, and each of these smaller divisions indicates, with respect to the pointer that rotates twenty-four times a day, one sixtieth of one twenty-fourth of a day or one minute. Hence this pointer is called the "minute hand." Similarly, the third pointer, when used, is the "second hand."

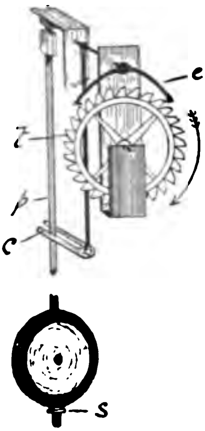

FIG. 75.

We cannot here enter upon a description of the mechanism and mode of action of a clock. It must suffice to give a very simple outline of the way in which the required regularity in the utilisation of the energy stored in the raised weight or the coiled spring is secured. Briefly, this is done, in clocks, by taking advantage of the property of a pendulum which we have already mentioned, namely, that so long as its length is unaltered its oscillations are always performed in the same time. It was Galileo who, about 1583, first attempted the construction of a clock with a pendulum, and his attempt was followed by many others, but it was not until 1657 that Huygens, a Danish man of science, really solved the problem in a practical way. Successive improvements have been made since his time, but the leading idea remains the same. This is illustrated diagrammatically in *Fig.* 75. The pendulum *p* (whose length can be varied as desired by means of the screw *s*) transmits, through the clutch *c*, a rocking motion to a curved piece of metal *e*, which has two projections at its ends so placed as to engage alternately the teeth of the wheel *t*. As the pendulum swings towards the right the projection on that side is raised, one of the teeth is released, and the wheel moves forward. Immediately, however, the projection on the left, descending, catches another tooth and stops the wheel. On the return swing the projection on the left is raised, and releases the wheel, which moves forward again, only to be stopped once more by the descent of the projection on the right. The arrangement is such that the wheel moves forward by the width of one tooth for each complete oscillation (see § 165) of the pendulum. It is plain that if the size of the wheel and the width of the teeth be adjusted so as to suit the length of the pendulum, the wheel can be made to advance an amount corresponding to one second at each movement. It should be mentioned also that the shape of the projections and that of the teeth are so adapted that each time the wheel is released a slight impulse is given to the pendulum, which suffices to prevent its stopping, as it would otherwise do, after a while, owing to the resistance of the air, friction, etc. Such an arrangement as is here indicated is called an escapement. The

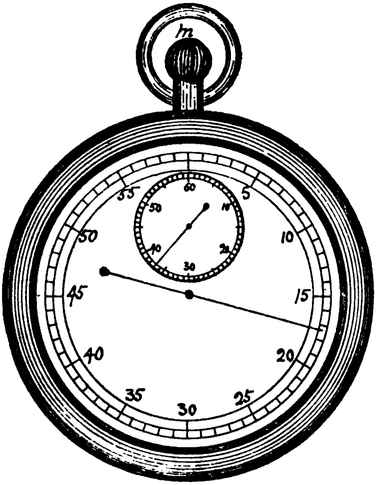

FIG. 76.

escapement-wheel controls the movements of all the other wheels, which can rotate only while the former is free.

In the watch the place of the pendulum is taken by the so-called balance wheel, but the underlying principle is just the same.

In the measurements of time with which we are here concerned we shall need to be able to mark the beginning and end of a definite interval of time, and to know the number of seconds contained in that interval. For this purpose we shall make use of a watch specially constructed so that it records minutes and seconds, and can be started or stopped at the will of the user. Such an instrument is known as a **stop-watch.** A very convenient form is that shown in *Fig.* 76. The small hand moves over a circle marked with 60 divisions, and makes a complete circuit in one hour, thus recording minutes. The large hand moves over a circle with 60 larger and 300 smaller divisions and makes a complete circuit in one minute. (In *Fig.* 76 the smaller divisions are not marked.) It thus records intervals of one-fifth of a second. The watch is wound by turning the milled-head *m*. This milled-head can also be pressed down, and by its depression three different effects can be produced. One depression "sets" the watch—that is to say, brings the two pointers to the zero marks and leaves them stationary there. A second depression starts the watch. A third depression stops the watch, the position of the hands marking, to 0.2 *sec.*, the duration of the interval between the starting and stopping.

§ 168. RATE OF A WATCH.—If we were sure that a watch, such as the stop-watch just described, really marked mean solar seconds, it would be a mere question of skill in starting and stopping it to determine the duration of an interval of time with an accuracy limited only by the delicacy of the instrument. But there are many causes which contribute to render the indications of a watch untrustworthy. Changes of temperature cause the various working parts to expand or contract, and though attempts are made in the construction of all good watches and clocks to nullify this effect, by making the expansion of one part counteract that of another, it is almost impossible entirely to get rid of it. Moreover, when, as is always the case in a watch, the motive power is the energy of a coiled spring, it is hard to avoid the difficulty that as this spring unwinds, as the watch runs down in fact, the impulse which it gives to the wheels becomes less and less, and their movements gradually become slower. This difficulty it is sought to meet also by peculiarities in construction, but it is only in a large and particularly good form of watch, known as a chronometer (from the Greek words for "time" and "measure"), that a very close approximation to uniformity is obtained. In any ordinary watch there are irregularities in the movements which are easily perceptible.

Now, if the duration of a second as indicated by a clock or watch be not actually that of the fraction of a mean solar day to which the name second is properly applied, it is plain that the instrument will, as we commonly say, lose or gain: it will indicate the lapse of either more or less than twenty-four hours in a mean solar day. The daily loss or gain is called the *rate*. The accurate determination of this rate involves certain astronomical observations, and is a matter of great importance in certain cases, particularly in those of chronometers which are used in navigation.

But from what has been said above, it will be plain that the loss or gain, which in the course of a day amounts to the interval called the rate of the watch, may not be, and probably is not, uniformly distributed throughout the whole day, and where short intervals of time have to be measured it becomes important to know what is the actual loss or gain in those intervals. This may be determined by comparing the indications of the watch with those of a seconds-pendulum. For this purpose evidently a very carefully constructed pendulum of exactly the right length is needed, but we can illustrate the method, even though the result will not be so trustworthy as we might desire, by means of the seconds-pendulum which we have already set up, and which we will assume to be correct.

Exercise LXX.

To determine, approximately, the rate of a stop-watch during a short interval of time.

Apparatus.—Stop-watch, seconds-pendulum (best of the form described in the note to § 166, but failing that, the one set up in Exercise LXIX., reading-telescope (§ 139), sheet of white paper or cardboard.

Method.—The sheet of paper or cardboard is supported vertically, a little way behind the lower end of the pendulum. The reading-telescope is set up several metres away, and its eye-piece is carefully focussed upon the cross-wires, which are turned so that one of them is vertical. Then the telescope itself is focussed upon the suspending-thread of the pendulum, a few *cms.* above the bob, so that the vertical cross-wire is exactly upon the thread. The pendulum is set swinging through a small distance (not more than 3 or 4 *cms.* to either side of the vertical), care being taken that the bob moves parallel to the paper or card behind it, and does not describe a closed curve. On looking through the telescope the suspending thread is seen to move to and fro across the field of vision, being twice in each complete oscillation momentarily hidden behind the vertical cross-wire. These momentary eclipses of the thread by the cross-wire are very sharp and well-defined, and serve for counting the oscillations. In doing this only the passages in one direction, say left to right, are to be counted. In counting it is better to begin a little before the passage which is to be taken as the starting-point, thus: "3, 2, 1, *o*, *1*, *2*, etc.," the starting-point being named *o*, because what we wish to know is the number of *intervals* between the passages. The stop-watch is wound (*see* note below) and set, care being taken to see whether there is any error, that is, whether the hands really come back to zero, and to note any correction which may be needed (see § 29). The observer then seats himself at the telescope, holding the stop-watch, resting upon the table, with the right hand, the forefinger touching the top of the milled-head. He begins to count, as above, "3, 2, 1, *o*," and exactly at the moment of the passage *o* presses down the milled-head so as to start the watch. The counting is continued, the passages in the same direction as that named *o* only being reckoned. The watch meanwhile is laid down, and not held in the hand, or it will become warmed, and its rate will alter. When the counting has reached *40* the watch is again raised, and the observer prepares to stop it. At the precise moment of the passage counted *50* the milled-head is pressed down and the watch stopped. The number of minutes, seconds, and fifths of a second is read and recorded, any necessary correction being added. This gives the time, to 0.2 *sec.*, recorded by the watch during 50 complete oscillations of the pendulum.

The watch is then set again, and at least one more count made in exactly the same way.

There must not be a difference of more than 0.4 *sec.* between the whole times recorded for any two counts of 50 complete oscillations. Should any greater difference than this appear it may be due either to a miscount or to some delay in starting or stopping the watch. Whatever the cause, one of the readings must be rejected, and a fresh count must be made to determine which.

Take the mean (§ 17) of the times found for 50 oscillations, and thence calculate the mean time recorded by the watch as that of one oscillation. It is manifest that, assuming the correctness of our seconds-pendulum, this ought to be 2 *secs.* From the actual result calculate the amount gained or lost by the watch per second.

[*Note.*—The reason why the watch is wound is that the observations may be made when a definite portion of the spring is in use, since, as indicated above, the "rate" for short periods alters a little as the spring unwinds.]

§ 169. EASY METHOD OF COUNTING OSCILLATIONS.—The error made in observing the whole time of any number of oscillations of a pendulum in the manner described above need not exceed 0.4 *sec.*, and will probably be about the same whether the number counted be twenty or two hundred. It is evident then that the larger the number of oscillations counted the less is the influence of this error upon the time deduced for one oscillation. It would have been preferable, in the last exercise, to have taken two or three counts of 100 oscillations, but this would be a very tedious operation. Fortunately, it is possible to count large numbers in a much easier way.

Suppose we have a pendulum which is, say, approximately the right length for a seconds-pendulum, and have determined, as above, the mean time of 50 complete oscillations. Let us assume that in so doing we have made an error of 0.4 *sec.* Then the error on our estimate of the time of one vibration is 0.008 *sec.*

Now let the pendulum be set swinging, and let the stop-watch be started precisely at a moment when the thread is passing behind the cross-wire from left to right. If the pendulum be now left to itself for five minutes or so it will perform somewhere about 150 oscillations, and if the watch be then stopped at another moment when the thread is passing behind the cross-wire from left to right *the time recorded will be that of some whole number of oscillations.* We can find this number by taking advantage of the fact that we already know very nearly the time of one oscillation. If we divide the observed interval by the approximate time of one oscillation we shall obtain a number which, while probably not whole, is yet within one unit of the actual number of oscillations performed. For supposing the error of our original estimate to have been 0.008 *sec.*, this would only amount on 150 oscillations to 1.2 *secs.*, which is less than the time of one oscillation. Hence the *nearest whole number* to that found by dividing the approximate time of one oscillation into the time for which the pendulum was allowed to swing between the two observations, will be the number of oscillations which it made. From these data a new, and more accurate, estimate of the time of a single oscillation can be obtained.

Take, for example the following case:—

Observed mean time of 50 oscillations = 101.2 *secs.* Whence mean time of one oscillation = 2.024 *secs.* The pendulum was set swinging, and the time registered by the watch between two particular passages in the same direction was 5 *mins.* 7.6 *secs.*

$$\text{The number of oscillations} = \frac{\text{whole time}}{\text{time of 1 oscillation}}$$

$$= \frac{307.6 \textit{ secs.}}{2.024 \textit{ secs.}} = 151.97$$

The nearest whole number is 152.

Hence 152 oscillations were performed in 307.6 *secs.*, and the time of 1 oscillation is 2.0236 *secs.*

Since the error in the whole time should not exceed 0.4 *sec.*, this value should be accurate to within 0.003 *sec.*

Again the pendulum was allowed to swing, and the interval between two successive passages in the same direction was 13 *mins.* 29.4 *secs.*

As before the number of oscillations is

$$\frac{809.4 \textit{ secs.}}{2.0236 \textit{ secs.}} = 399.98$$

The nearest whole number is 400.

Hence 400 oscillations have occurred in 809.4 *secs.*, and the time of 1 oscillation is 2.0235 *secs.* Assuming that the whole time is not more than 0.4 *sec.* out, it follows that this value for the time of a single oscillation will not differ from the true one by more than 0.001 *sec.*

The student should now make use of this method to get an estimate of the rate of a stop-watch as compared with seconds-pendulum during, say, a quarter of an hour.

Exercise LXXI.

To determine to a nearer approximation the rate of the stop-watch, the seconds-pendulum being assumed correct.

Apparatus.—As for Exercise LXX. (§ 168).

Method.—Use the approximate time of 1 oscillation found in Exercise LXX. Wind and set the stop-watch. Start it at the precise moment of a passage of the suspending thread behind the cross-wire from left to right, exactly as directed in Exercise LXX. Let the pendulum swing for about 5 minutes. Stop the watch again exactly at the moment of a left-to-right passage. Calculate, as above, the number of oscillations, and from this, and the whole time, deduce the time of 1 oscillation. Using this new value repeat the process, allowing about a quarter of an hour between the observations.

[*Note.*—The student should make quite sure that he understands the principle of this method before he uses it. Why is a short interval taken first and then a longer one? What would be the objection to taking a long interval at once?]

§ 170. RELATION BETWEEN LENGTH AND TIME OF OSCILLATION OF A SIMPLE PENDULUM.—It was stated, in § 165, that the time of oscillation of a pendulum depends upon its length, and we have seen that by using a pendulum of particular length we can reproduce, so to speak, the standard second. We have now learned how the oscillations of a pendulum are observed, how the time in which they are performed can be determined with the help of a stop-watch, and how, finally, it is possible, without an unduly prolonged effort of attention, to count very large numbers of oscillations. We are therefore in a position to investigate the relation between the length of a pendulum, like that of *Fig.* 72, and the time in which it performs an oscillation.

To do this we need only modify very slightly the apparatus of *Fig.* 72 (§ 166). The suspending thread, after passing through the eye of the needle and being drawn through the knife-cut in the cork, is, instead of being attached to the cork with sealing-wax, simply tied to a clamp or ring fixed upon the retort-stand above the one which carries the cork and needle and projecting slightly behind it. The length can then be altered at will by raising or lowering the clamp carrying the needle.

The measurement of the length can be carried out after the observations of the time of oscillation, by bringing the lower clamp into position under the bob, raising it so that the top of the screw *nearly* touches, making it fast, and completing the adjustment by turning the screw itself until the bob just grazes. Then, the suspending thread being disconnected at the top, the pendulum can be removed and the distance from the top of the screw to the needle's eye measured with a metre scale, or, if greater than a metre, accurately copied upon the wooden rod used in Exercise LXIX. and then measured. This distance, diminished by the radius of the bob, gives the length of the pendulum.

Rough observations with the naked eye will be sufficient to show that the oscillations become more rapid as the pendulum is shortened, but that the halving of the length only reduces the time of an oscillation by considerably less than half.

Theoretical considerations, into which we cannot here enter, indicate that if all the weight of a pendulum were concentrated at the centre of the bob, and if the suspending thread were perfectly rigid as well as devoid of weight, then the reduction of its length to one quarter should reduce the time of an oscillation to one-half of its former value, the quadrupling of the length should make the time of an oscillation twice as great; in short, that the time of an oscillation should vary as the square root of the length of the pendulum.

A pendulum fulfilling these ideal conditions is called a "simple pendulum." It is manifest that the pendulum we are about to use is not, in this sense, "simple," but we may expect to find this relation between length and time of oscillation approximately true, even though the conditions are not exactly fulfilled.

Exercise LXXII.

To determine the relation between the length and time of oscillation of an approximately simple pendulum.

Apparatus.—Same as for Exercise LXIX., with additional clamp as described above; also same as for Exercise LXX.

Method.—Set up the pendulum by threading the suspending fibre through the eye of the needle, drawing it into the knife-cut in the cork, and making it fast to the clamp above and behind, giving it any convenient length, but preferably less than 1 metre over all for convenience in measuring. Having arranged and focussed the reading-telescope, find exactly, as in Exercise LXX., the mean time of 50 oscillations, or, if the oscillations be fairly rapid, that of 100. Deduce the approximate time of one oscillation. Use this, as in Exercise LXXI., to count the oscillations during a longer interval, bearing in mind the answers to the questions asked in the note to § 169. Having found the time of one oscillation within 0.001 *sec.* adjust the clamp and then the screw at the bottom so as just to graze the bob. Release the suspending thread at the top. Remove the pendulum. Measure the distance from the flattened top of the screw to the needle's eye. Subtract from it the radius of the bob, as deduced from the mean diameter found with the slide callipers.

Record the difference as the length of the pendulum.

Make three sets of observations like this with three different lengths.

Taking the times indicated by the stop-watch as accurate (unless the watch is known to have a seriously large rate) record your results thus:—

Number of Experiment.	Length of Pendulum in *mms.* (l)	Time of one Oscillation in *secs.* (t)	Value of $\frac{t^2}{l}$
1			
2			

etc.

What is the mean of the numbers in the last column? What is the greatest percentage difference between any one of them and the mean? Do your experiments justify the description of the pendulum used as "approximately simple," if it be taken for granted that the time of oscillation of a "simple pendulum" is proportional to the square root of its length?

[*Note.*—The student should bear in mind that the data he possesses concerning his seconds-pendulum, namely, its length as set up and the time of its oscillation as determined in Exercise LXXI., furnish a fourth pair of values for comparison with the three obtained above.]

ANSWERS TO PROBLEMS

No.

Page 30

1. 10 : 1 nearly.
2. 0.2 in.
3. 4810.4 metres.

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4. (*a*) $\frac{1}{50}$. (*b*) 0.01. (*c*) 24.5 ins.
5. For No. 1 vernier length equal to 24 scale divisions (=6 cms.); for No. 2 vernier length equal to 26 scale divisions (=6.5 cms.) divided in each case into 25 equal parts.

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6. If length of side be l cms. the area is $6l^2$ sq. cms.
7. The rectangular block has a greater surface by 0.011 sq. cm.
8. (*a*) 62.352 sq. cms. (*b*) 20.784 cms.

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9. No.
10. Yes. 2.6 c.cs.

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11. 11.4.
12. 0.915.
13. 5 grms.

Page 169

14. 111.24 lb.-weight.
15. 10 cms.
16. 1033 grms.-wt. per sq. cm. (nearly).
17. 26 cms.

No.

Page 180

18. (*a*) 3.4 c.cs. (*b*) 34 grms.
19. (*a*) 15 c.cs. (*b*) 0.8 grms. per c.c.
20. Equal.

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21. (*a*) 7.5. (*b*) 468.75 lb. per. cub. ft.
22. No.
23. 21.35.

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24. 0.85 kilogrms. per litre.
25. $1.2\dot{6}$.

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26. 0.25.

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27. 0.84.

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28. (*a*) 4. (*b*) 0.0009 cub. ft. (*c*) 4000 ozs. per cub. ft.
29. 25 grms.
30. See § 146.
31. 1.3.

Page 198

32. No. See sections quoted, and especially Fig. 57, p. 167.
33. 1000 grms.-wt. per sq. cm.

Page 199

34. 802.56 cms.
35. 2.4 feet.

Page 204

36. (*a*) 22.8 litres. (*b*) 85.5 cms.

No. **Page 207**

37. 100 c.cs.

Page 208

38. 20 c.cs.
39. (*a*) 139 c.cs. (*b*) 135 c.cs.

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40. (*a*) 750 mms. (*b*) 840 mms.
41. 21 cms. of mercury.

No. **Page 210**

42. 53.05 c.cs.

Page 211

43. 141.96 c.cs.
44. $668.\dot{5}7142\dot{8}$.
45. 37° C.
46. See section quoted (§ 155, p. 208).

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47. 12.288 ins.
48. 2 litres.

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